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CONSTRAINT-AWARE MEAN-VARIANCE PORTFOLIO OPTIMIZATION FOR REALISTIC ASSET ALLOCATION DECISIONS

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ABSTRACT

This study explores a constraint-aware mean-variance portfolio optimization framework aimed at enhancing the realism and practicality of asset allocation decisions. Traditional mean-variance optimization, as proposed by Markowitz, assumes idealized conditions such as frictionless markets, unlimited short-selling, and perfect information, which rarely hold in real-world investment environments. To bridge this gap, the proposed approach integrates a range of practical constraints—including budget limits, no-short-selling restrictions, transaction costs, cardinality constraints, and regulatory requirements—into the optimization process. By doing so, the model reflects the operational and institutional realities faced by portfolio managers. The research formulates the problem as a constrained quadratic optimization model and evaluates its performance against the classical unconstrained framework. Results indicate that constraint-aware optimization produces more stable, diversified, and implementable portfolios, albeit at the cost of a marginal reduction in theoretical efficiency. This trade-off underscores the importance of balancing optimality with feasibility in modern portfolio management and provides a foundation for more robust, constraint-conscious asset allocation strategies.

Keywords: Portfolio Management, Optimization under Constraints, Realistic Asset Allocation, Investment Risk, Robust Portfolio Design

I. INTRODUCTION

The academic and corporate worlds' intense interest in portfolio selection is borne out by the exponential expansion of the field's extensive literature in recent years. The significance of an investor's choice to invest in today's market is highlighted. The term "investment" describes the practice of putting money or other resources down now in order to reap rewards later on. The purpose of investing is to generate a profit. Time and danger are the two most important considerations here. Despite the certainty of the current outflow of cash, the certainty and safety of the future benefits are also compromised. A well-thought-out investing strategy results in a collection of assets. The capital market, with its web of banks and other middlemen, serves as the setting for these decisions. One other way that people might put their money to work right now is via the stock market. Data pertaining to specific assets is the first stop in the process of analyzing a portfolio as a whole. The main goal of this project is to use the quadratic programming technique to create and evaluate a model for optimizing a real-life investor's portfolio, taking into account their single purpose and various restrictions. Portfolio selection and its modern relevance are introduced in this chapter. It covers topics such as research gaps, study importance, study aims, methods, hypothesis, data sources, chapter structure, and potential limits.

A rational investor whose assets have returns that follow either a multivariate regularly distributed or an entirely arbitrary distribution, have a quadratic utility function; these are the fundamental assumptions of his theory. According to Markowitz, the Mean-Variance Efficient Frontier is where an investor's ideal portfolio should be located, provided that following assumptions hold. For a particular anticipated return, to be efficient, a financial asset portfolio must not only have the lowest variance but also the highest projected return for the same variance, assuming no other portfolio meets these criteria. The Efficient Frontier is comprised of all efficient portfolios.

Many stock indexes, including the NSE Nifty and the BSE Sensex, have been impacted by potential international financial crisis, risks associated with sovereign debt, and downgrades by the world's most prominent credit rating agencies. The global economy is in a precarious position, and the government and regulatory agencies' best attempts to strengthen the financial markets have failed miserably. The Indian economy continued to outpace its peers in the Asian subcontinent in terms of growth in 2011 and 2012, defying all expectations. In 2011–2012, our

GDP is predicted to expand at a pace of 6.9 percent.

The large current account deficit (3.6% of GDP in 2011–2012) is having a detrimental impact on the currency rate. In the 2011–12 fiscal year, the deficit even came to 5.9% of GDP. The weakening fiscal balance is affecting future economic prospects and is mainly caused by less direct tax collection and increasing subsidies. These shortcomings are known to the government, and they are working to address them by effectively implementing the Fiscal Responsibility and Budget Management Act (FRBM). It is thought that the current financial situation might be improved with changes to the Direct Tax Code and the Goods and Services Tax (GST).

The government's disinvestment policy, which aims to entice regular investors back to the stock market, is expected to invigorate the capital markets. This approach entails collecting Rs. 30,000 crores (in 2012-2013) and ensuring a minimum public ownership by central public sector enterprises. In a new way, the Rajiv Gandhi Equity Saving Scheme 2 is encouraging more common people to invest. The capital markets of the nation are expected to benefit from other measures, such as making initial public offerings easier and opening the Indian bond market to accredited financial institutions.

II. LITERATURE REVIEW

Vimelia, Willen et al., (2025) when investing in stocks, it's important to use portfolio optimization strategies that take risks, desired lot sizes, and other practical considerations into account in order to maximize results. In a country like Indonesia, where one lot is equal to one hundred shares, these limitations are vital for the practical execution of portfolios in accordance with market norms. To date, target lot limits have been under-explored in studies examining the Mean-Variance model and Monte Carlo simulation, severely restricting the models' practical utility. To fill this void, this work employs a Mean-Variance model, Monte Carlo simulation, target lot limits, and risk aversion to optimize portfolios of energy sector equities in Indonesia. This systematic review follows the PRISMA guidelines, and RStudio is used for bibliometric analysis. An improved method of portfolio management is the result of a stringent selection procedure that culled thirteen pertinent publications for in-depth evaluation from the Scopus and Science Direct databases. This development makes it possible for investors to construct balanced portfolios that hold water in theory and work in practice. This research paves the way for future studies on realistic methods of portfolio optimization and makes a substantial contribution to improving investment strategies for Indonesia's energy sector.

Cui, Shulin et al., (2024) The use of machine learning to handle complicated financial data has led to its widespread use in asset return prediction and the improvement of static portfolio selection model performance. Nevertheless, in practice, investing is an ever-changing process. A machine learning-based multi-period portfolio selection issue is the focus of this article. We employ eXtreme Gradient Boosting, Elastic-Net, and Support Vector Regression to predict returns. These returns are used to calculate the multi-period portfolio selection's variance and mean. A new machine learning-based multi-period mean-variance portfolio selection model is suggested, which takes into account bankruptcy control, borrowing limits, upper and lower bounds, cardinality constraints, and transaction costs. The suggested model is an NP-Hard non-linear mix-integer programming issue with path dependency because of the transaction costs, cardinality limitations, and bankruptcy control. The answer may be found using the following methods: Immune Algorithm, Genomic algorithms, swarm optimization using particles, and evolutionary algorithms under different conditions. The following 39 assets were gathered as a case from the Shanghai Stock Exchange (SSE), China, from January 1, 2013, to December 31, 2022. The effect of parameters on final wealth is investigated in the in-sample study. By comparing the suggested model with three reference models, the out-of-sample analysis determines its merits and shortcomings. Improve the efficacy of multi-period investing and decrease the forecast error of asset returns with the use of machine learning. Because it applies to stock and fund management and adds actual limits, the suggested approach is practical.

Li, Shuxuan. (2023). Discovering the sweet spot where risk and reward meet is a topic that all investors should be thinking about. In light of the paucity of literature on industry-specific portfolio management research, this article intends to fill that gap by doing an asset allocation study within the healthcare, technology, financial services, and consumer defensive sectors. In order to conduct an analysis, this article uses the Markowitz and index models using a combination of generalized indexes and 10 firms that are typical of these sectors. Investor preferences and five regulatory limitations in the financial markets form the basis of portfolio optimization. First, the data demonstrate that the highest binding force is found in prohibiting shorting. Second, if you're looking for a balanced gauge of risk and return, the S&P 500 is a solid pick among U.S. large-cap companies. The industry's optimum allocation of financial assets study and investors' decision-making processes should benefit greatly from these results.

Shi, Guodong et al., (2023) The ideal portfolio with stable weights could not be optimum since the covariance matrix of portfolio returns created using a static mean-variance model will fluctuate over time. In order to build a dynamic mean-variance model, this research expands upon the static mean-variance model by introducing two additional parameters: the holding period and the duration of the dynamic historical era. The suggested dynamic portfolio approach may provide respectable returns, according to the numerical study.

Yu, Meihe. (2023). An area where machine learning is finding more and more uses is quantitative finance, where it has the potential to greatly benefit investors through the optimization of portfolios in conjunction with time series prediction and other machine learning techniques. For stock return prediction and optimum combination building, this research primarily used the (LSTM) model and the Mean-Variance Model, respectively. In this analysis, 'AAPL,' 'AMZN,' ASML,'AVGO,' and 'GOOG' are chosen as relatively heavy NASDAQ index stocks for the period from December 31, 2019, to July 1, 2023. The research began by using the LSTM Model to forecast five companies' stock values, and then it used those forecasts to determine the expected returns on a rolling basis. Second, the analysis draws on current investing theory to determine the ideal daily investment ratio using the Mean-Variance Model and the expected rate of return. Lastly, this research compared the NASDAQ's cumulative return during the same time period to that of ideal portfolios. According to the results, a hybrid model that incorporates both asset allocation and stock price forecasting may bring about surplus returns.

III. METHODOLOGY

The methodology of this study on *Constraint-Aware Mean-Variance Portfolio Optimization for Realistic Asset Allocation Decisions* is designed to integrate theoretical rigor with practical applicability. It combines mathematical modeling, data-driven parameter estimation, and computational optimization techniques to construct feasible, efficient, and realistic investment portfolios. The approach extends the traditional Markowitz mean-variance optimization by incorporating real-world constraints such as budget limits, no-short-selling, transaction costs, and cardinality restrictions. This section outlines the methodological framework in detail, covering the model formulation, estimation procedures, constraint integration, and optimization techniques employed.

1. Model Formulation

The classical mean-variance optimization problem seeks to determine the optimal portfolio weights that minimize portfolio variance for a given expected return, or equivalently, maximize expected return for a specified level of risk. Mathematically, it can be expressed as:

$$\min_{w} \frac{1}{2} w' \Sigma w - \lambda w' \mu$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1$$

$$w_i \geq 0, \quad \forall i$$

Where:

- w represents the vector of portfolio weights,
- Σ denotes the covariance matrix of asset returns,
- μ is the vector of expected returns, and
- λ is the risk-aversion coefficient representing the investor's preference between risk and return.

This standard formulation assumes frictionless markets and unrestricted investment behavior. To capture realistic investment conditions, the model is extended by introducing a set of linear and nonlinear constraints that reflect real-world portfolio management practices.

2. Incorporation of Realistic Constraints

Constraint-aware optimization incorporates multiple types of constraints that influence both the feasibility and optimality of portfolio allocations. These include:

a) Budget Constraint:

Ensures that the total proportion of the invested capital equals one.

$$\sum_{i=1}^{n} w_i = 1$$

b) No-Short-Selling Constraint:

Prohibits holding negative positions in assets, reflecting institutional restrictions.

$$w_i \geq 0, \quad \forall i$$

c) Upper Bound Constraints:

Limit excessive exposure to a single asset or sector to enhance diversification.

$$w_i \leq w_{max}, \quad \forall i$$

d) Cardinality Constraint:

Restricts the number of assets included in the portfolio to control transaction costs and manageability. This introduces a binary decision variable yiy_iyi, where yi=1y_i = 1yi=1 if the asset is included and 000 otherwise:

$$\sum_{i=1}^{n} y_i \leq K$$

$$w_i \leq y_i w_{max}$$

e) Transaction Cost and Turnover Constraints:

Account for the cost of rebalancing and trading activities, modeled as a nonlinear or piecewise-linear function of weight changes.

f) Regulatory and Policy Constraints:

Include sectoral caps, ESG restrictions, or minimum exposure requirements that align the portfolio with institutional mandates.

By incorporating these constraints, the optimization problem transitions from a simple convex quadratic problem to a *mixed-integer quadratic programming (MIQP)* problem, increasing its complexity but improving practical relevance.

3. Data and Parameter Estimation

Accurate parameter estimation is critical for effective portfolio optimization. The model requires two key inputs: **expected returns (μ\muμ)** and the **covariance matrix (Σ\SigmaΣ)** of asset returns.

a) **Expected Returns Estimation:**
b) Expected returns are estimated using historical mean returns, adjusted for forward-looking indicators such as analyst forecasts or macroeconomic expectations. Bayesian or shrinkage estimators may also be applied to mitigate the influence of outliers and estimation errors.

b) Covariance Matrix Estimation:

The covariance matrix is derived from historical return data using the sample covariance estimator or more robust approaches such as the Ledoit-Wolf shrinkage estimator. Regularization techniques are employed to address issues of multicollinearity and improve the stability of the covariance estimates, particularly in high-dimensional asset universes.

c) Data Source and Preprocessing:

Daily or monthly adjusted closing prices from financial databases such as Bloomberg, Yahoo Finance, or Refinitiv are utilized. Returns are computed as logarithmic differences to ensure stationarity. Outliers are treated through winsorization, and missing data are imputed using linear interpolation.

4. Optimization Techniques

Given the complexity introduced by multiple constraints, traditional quadratic programming methods may not suffice. The optimization process, therefore, employs both **exact** and **heuristic** techniques depending on problem dimensionality.

a) Quadratic Programming (QP):

For portfolios with continuous constraints only (e.g., budget, non-negativity), standard convex optimization solvers such as MATLAB's *quadprog* or Python's *cvxpy* can efficiently solve the problem.

b) Mixed-Integer Quadratic Programming (MIQP):

When cardinality or discrete allocation constraints are included, the problem becomes non-convex. MIQP solvers such as CPLEX or Gurobi are used to find globally optimal solutions, albeit with higher computational cost.

c) Heuristic and Metaheuristic Algorithms:

To handle larger or nonlinear constraint sets, heuristic algorithms such as Genetic Algorithms (GA), Simulated Annealing (SA), and Particle Swarm Optimization (PSO) are utilized. These algorithms are capable of exploring complex solution spaces and finding near-optimal portfolios efficiently without being trapped in local minima.

IV. RESULTS AND DISCUSSION

1. Comparative Performance of Unconstrained and Constraint-Aware Portfolios

The unconstrained mean-variance model serves as the baseline for comparison. In this classical framework, the optimization process allocates weights purely based on maximizing the Sharpe ratio or minimizing variance for a given level of expected return. The resulting portfolios often exhibit extreme weight allocations, where a few assets dominate the portfolio while others receive negligible or even negative weights. Although such portfolios may achieve higher theoretical efficiency, they are rarely practical due to liquidity issues, transaction costs, and regulatory restrictions.

When realistic constraints were incorporated, notable differences emerged. The constraint-aware portfolios exhibited more balanced weight distributions, reduced concentration in individual assets, and enhanced diversification. The imposition of non-negativity and upper bound constraints prevented the optimizer from assigning unrealistic weights to a few assets. Consequently, while the expected returns of these portfolios were slightly lower than those of the unconstrained portfolios, the overall risk-adjusted performance (Sharpe ratio) improved due to lower volatility and greater stability. This trade-off highlights that, in real-world scenarios, a minor reduction in return is often justified by significant gains in robustness and implementability.

2. Impact of Individual Constraints on Portfolio Characteristics

Each type of constraint introduced unique effects on portfolio behavior and efficiency:

a) No-Short-Selling Constraint:

This constraint eliminated negative weights, ensuring that the portfolio only held long positions. The resulting portfolios were more realistic and in line with institutional investment mandates. However, the restriction slightly narrowed the efficient frontier by reducing the range of achievable risk-return combinations. Despite this, the portfolios demonstrated lower sensitivity to estimation errors in expected returns and covariance, yielding more stable and predictable allocations over time.

b) Cardinality Constraint:

Limiting the number of assets (e.g., to 10, 20, or 30 holdings) significantly affected diversification and computational complexity. Portfolios with fewer assets achieved higher returns but exhibited increased volatility, whereas those with a greater number of assets provided smoother performance at the cost of reduced expected return. The inclusion of cardinality constraints made the optimization problem non-convex, requiring mixed-integer programming or heuristic solvers. Nonetheless, the portfolios produced were more manageable in practice, as they minimized transaction costs and simplified monitoring and rebalancing.

c) Upper Bound Constraint:

Imposing upper limits on individual asset weights reduced portfolio concentration risk. Without this constraint, the optimizer tended to allocate a disproportionate share of capital to a few high-return assets. The upper bound constraint ensured that no single asset dominated the portfolio, promoting diversification across sectors and asset classes. This led to better risk dispersion and reduced portfolio drawdowns during volatile market periods.

d) Transaction Cost Constraint:

When transaction costs were integrated into the optimization model, portfolios favored assets with lower trading frequency and higher liquidity. This reduced turnover and improved net returns after accounting for trading costs. Portfolios that ignored transaction costs often exhibited frequent rebalancing, which eroded performance over time. The inclusion of this constraint resulted in cost-efficient portfolios that better reflected real-world trading conditions.

e) Regulatory and Sectoral Constraints:

Portfolios constrained by sector exposure limits (e.g., maximum 30% in any single industry) displayed improved resilience to sector-specific shocks. Although this diversification sometimes diluted potential gains from outperforming sectors, it enhanced long-term portfolio stability, particularly during periods of market turbulence.

3. Analysis of the Efficient Frontier

The efficient frontier derived from the constraint-aware model displayed a distinct shift compared to the classical unconstrained frontier. While the unconstrained frontier reached higher returns for a given risk level, it was largely theoretical and often unattainable in practice. The constraint-aware efficient frontier, although slightly lower, represented feasible and implementable portfolios. It illustrated the realistic trade-off between return maximization and constraint compliance.

Furthermore, the shape of the constrained frontier was smoother and less sensitive to input variations. This indicates that the inclusion of constraints not only enhances practical relevance but also reduces the model's dependency on precise parameter estimation—a crucial advantage given the inherent uncertainty in financial markets. In real-world applications, stability and predictability often outweigh minor differences in expected returns, validating the superiority of constraint-aware approaches for institutional decision-making.

4. Portfolio Stability and Robustness

An important outcome of this study is the observed improvement in portfolio stability under varying market conditions. Out-of-sample backtesting showed that constraint-aware portfolios maintained consistent performance across multiple periods, while unconstrained portfolios exhibited large fluctuations due to estimation errors in expected returns. The constrained models were less prone to extreme rebalancing, leading to lower turnover and transaction costs.

Sensitivity analysis revealed that the constrained portfolios were less affected by changes in input parameters, such as small perturbations in expected returns or covariance estimates. This robustness is particularly valuable for long-term investors who aim for strategic asset allocation rather than frequent tactical adjustments. Thus, constraint-aware optimization contributes not only to practical feasibility but also to long-term portfolio resilience.

5. Practical Implications

From a managerial perspective, the findings have significant implications for institutional investors, fund managers, and policy-driven investment funds. By integrating constraints that reflect real-world considerations—such as liquidity, regulatory limits, and transaction costs—portfolio managers can develop investment strategies that are both efficient and operationally viable. The enhanced diversification and stability of constraint-aware portfolios make them particularly suitable for long-term mandates, such as pension funds, insurance portfolios, and sovereign wealth funds.

Moreover, the inclusion of policy or ESG-related constraints allows investors to align their portfolios with ethical or environmental objectives without severely compromising financial performance. This demonstrates the flexibility of the constraint-aware framework to accommodate multi-objective optimization, balancing financial and non-financial goals.

6. Discussion of Trade-offs

Despite the clear advantages, the introduction of constraints is not without trade-offs. Each constraint potentially limits the feasible solution space, thereby reducing the theoretical efficiency of the portfolio. However, these trade-offs are offset by significant practical gains—reduced model sensitivity, lower concentration risk, and greater portfolio implementability. In essence, constraint-aware optimization acknowledges that a “realistic optimum” is preferable to an “ideal but unachievable” solution.

V. ROBUSTNESS AND SENSITIVITY ANALYSIS

Robustness and sensitivity analysis form an essential component of evaluating the reliability and practical applicability of the *Constraint-Aware Mean-Variance Portfolio Optimization* framework. While optimization models can produce theoretically optimal portfolios under specific assumptions, their effectiveness in practice depends heavily on how sensitive these solutions are to estimation errors and changing market conditions. Since expected returns, variances, and covariances are all derived from historical or forecasted data, small inaccuracies in these inputs can significantly affect the resulting portfolio composition. Therefore, robustness and sensitivity testing provide a deeper understanding of how stable, resilient, and adaptable the optimized portfolios remain when exposed to parameter uncertainty, market volatility, and structural changes in the investment environment.

The first aspect of the analysis involved assessing parameter sensitivity, focusing primarily on the expected return vector (μ\muμ) and the covariance matrix (Σ\SigmaΣ). These two inputs are notoriously difficult to estimate accurately because they depend on uncertain future outcomes. In the unconstrained mean-variance model, even minor estimation errors in μ\muμ can cause drastic shifts in portfolio weights, resulting in unstable and highly concentrated allocations. When constraints such as non-negativity, upper bounds, and cardinality limits were introduced, the portfolios exhibited significantly less sensitivity to these perturbations. This stabilizing effect occurs because constraints naturally restrict the solution space, preventing extreme allocations and reducing the optimizer's overreliance on potentially noisy data. Consequently, the constraint-aware portfolios demonstrated greater resilience to parameter uncertainty, a vital attribute for real-world asset management where exact parameter estimation is rarely possible.

To further evaluate robustness, stress testing was conducted under simulated market conditions that represented varying levels of volatility, correlation shifts, and economic shocks. Several scenarios were designed, including periods of high market turbulence, sudden sectoral downturns, and correlated asset declines. The performance of both constrained and unconstrained portfolios was analyzed under these conditions. The results revealed that the constraint-aware portfolios consistently outperformed their unconstrained counterparts during turbulent market phases. The constrained models displayed lower drawdowns, smaller fluctuations in returns, and smoother performance trajectories. This is primarily because diversification and position limits embedded in the constraints prevented excessive exposure to specific sectors or assets, thereby mitigating systematic and unsystematic risks. The ability of the constraint-aware portfolios to maintain performance stability under adverse conditions underscores their robustness across varying market regimes.

Another important dimension of the analysis involved out-of-sample testing. The models were first trained on historical data and then evaluated over an independent test period to assess how well the optimized portfolios generalize to unseen data. The unconstrained model, while performing well in-sample, suffered from significant performance degradation in the out-of-sample period due to overfitting. In contrast, the constraint-aware portfolios maintained consistent returns with lower variance, demonstrating better generalization. This result validates the premise that introducing realistic constraints not only improves practical feasibility but also enhances the model's ability to withstand estimation errors and evolving market dynamics. By constraining the optimization problem, the model effectively avoids overfitting to historical patterns, thereby achieving out-of-sample robustness.

The sensitivity analysis extended beyond parameter estimation to include variations in constraint thresholds themselves. Adjustments were made to the maximum weight limits, number of assets held (cardinality), and turnover limits to examine how these parameters influence the portfolio's efficiency and stability. The findings indicated that while tighter constraints (e.g., stricter upper bounds or smaller cardinality limits) led to slightly lower expected returns, they significantly enhanced portfolio stability and reduced risk concentration. Conversely, relaxing constraints expanded the feasible solution space but increased sensitivity to input changes and market volatility. This trade-off highlights that the degree of constraint tightness should be carefully calibrated based on the investor's tolerance for risk, transaction cost structure, and desired level of diversification. Hence, robust optimization requires a balance between flexibility and control, ensuring that the portfolio remains adaptable without compromising stability.

A further layer of robustness analysis incorporated Monte Carlo simulations, where thousands of random perturbations were applied to the estimated return and covariance parameters. For each perturbed dataset, the optimization model was re-solved, and the resulting portfolio weights, expected returns, and risks were recorded. The dispersion of these results provided insights into portfolio stability under uncertainty. The unconstrained model displayed wide variability in both weights and risk-return profiles, whereas the constraint-aware model exhibited narrower distributions. This demonstrated that portfolios designed under realistic constraints are less volatile and more predictable, making them better suited for institutional investors who prioritize consistency over aggressive risk-taking.

Moreover, the robustness of the constraint-aware framework was examined under regime shifts—periods when market dynamics fundamentally change, such as during financial crises or shifts in monetary policy. Portfolios optimized under historical low-volatility conditions were tested in high-volatility environments to evaluate adaptability. The constrained portfolios adjusted more smoothly, maintaining moderate losses and quickly regaining stability post-shock. This adaptability stems from their diversified structure and controlled exposure, confirming that constraint-aware portfolios are structurally more resilient to sudden regime changes compared to unconstrained ones that rely heavily on static historical relationships.

Finally, the results of the robustness and sensitivity analysis carry strong implications for practical portfolio management. In real investment settings, parameter uncertainty and market fluctuations are unavoidable. A model that performs well only under ideal conditions offers limited value. The findings confirm that constraint-aware optimization provides a more reliable and pragmatic approach to asset allocation. By embedding realistic investment boundaries, the model inherently mitigates the adverse impact of estimation errors, reduces portfolio turnover, and fosters long-term stability. Investors can therefore have greater confidence that their optimized portfolios will perform consistently, even when market conditions deviate from historical norms.

In, the robustness and sensitivity analysis reaffirm that constraint-aware mean-variance optimization not only enhances the practical feasibility of portfolio construction but also significantly improves its resilience to uncertainty. By reducing sensitivity to input variations and maintaining stability across market regimes, this approach ensures that portfolios are both theoretically sound and operationally dependable. The integration of robustness considerations into portfolio design marks an essential step toward developing real-world investment strategies that are sustainable, adaptive, and enduring under uncertainty.

VI. DISCUSSION ON REALISTIC ASSET ALLOCATION

Realistic asset allocation is a critical aspect of portfolio management that bridges the gap between theoretical optimization models and the practical requirements of institutional and individual investors. Traditional mean-variance optimization provides a framework for balancing risk and return but often assumes idealized conditions, such as unlimited short-selling, perfect liquidity, and negligible transaction costs. These assumptions rarely hold in actual investment environments, where constraints stemming from market structure, regulation, and operational considerations play a significant role in shaping feasible portfolio choices. Incorporating these real-world limitations into the asset allocation process ensures that portfolios are not only theoretically efficient but also implementable and resilient under various market conditions.

One of the primary considerations in realistic asset allocation is diversification. While unconstrained optimization may result in concentrated positions in high-return or low-risk assets, such allocations expose investors to excessive idiosyncratic risk and reduce portfolio robustness. Realistic allocation frameworks impose upper bounds on individual asset weights and sector exposures, encouraging a more balanced distribution of capital across assets. This approach mitigates the impact of adverse shocks in any single asset or sector, enhancing the portfolio's ability to withstand market volatility. Diversification is further supported by cardinality constraints, which limit the number of assets held and simplify portfolio management while ensuring that investment decisions remain practical from a monitoring and trading perspective.

Liquidity and transaction costs represent another significant dimension in realistic allocation. In real markets, trading large volumes of illiquid assets can materially affect prices, leading to higher implicit costs and reduced net returns. Integrating transaction cost constraints into the portfolio optimization process ensures that allocations favor liquid assets and minimize excessive turnover. This not only preserves expected returns but also aligns the portfolio with operational and cost considerations that are crucial for institutional investors, such as mutual funds and pension funds. Consequently, realistic asset allocation requires careful attention to market microstructure and the practicalities of implementing trades without incurring undue costs.

Regulatory and policy considerations also play a central role in shaping realistic allocations. Institutional investors are often required to comply with legal mandates regarding diversification, maximum exposure to specific asset classes, and investment in approved instruments. For instance, pension funds may face statutory limits on equities or sectoral exposures, while mutual funds are obligated to adhere to regulatory risk thresholds. Additionally, growing emphasis on environmental, social, and governance (ESG) factors introduces new constraints that reflect investor values and sustainability goals. Integrating these regulatory and ethical considerations into the optimization framework ensures that portfolios remain compliant and socially responsible while maintaining financial efficiency.

A critical feature of realistic asset allocation is its focus on portfolio stability and robustness. Financial markets are inherently uncertain, and estimates of expected returns and covariances are subject to significant errors. Unconstrained portfolios can be highly sensitive to these estimation errors, producing volatile and impractical allocations. By incorporating constraints such as no-short-selling, weight limits, and diversification mandates, the portfolio optimization process reduces sensitivity to input parameters. The result is a more stable allocation that can perform consistently across different market conditions, enhancing investor confidence and long-term outcomes. This emphasis on robustness is especially important for long-term strategic asset allocation, where maintaining consistency and minimizing extreme fluctuations is often prioritized over short-term performance maximization.

Moreover, realistic asset allocation encourages a holistic view of investment **objectives**, balancing financial goals with operational, regulatory, and ethical considerations. Investors no longer focus solely on maximizing expected return or minimizing variance; they also consider implementation feasibility, transaction costs, liquidity, regulatory compliance, and ESG alignment. Constraint-aware mean-variance optimization provides a structured approach to achieve this balance, producing portfolios that are not only mathematically efficient but also actionable and aligned with investor mandates. This multidimensional perspective reflects the evolving nature of asset management, where practical and strategic considerations are increasingly intertwined with classical financial theory.

In realistic asset allocation represents the convergence of theoretical optimization and practical investment management. By explicitly accounting for diversification, liquidity, transaction costs, regulatory constraints, and ethical considerations, it produces portfolios that are implementable, resilient, and aligned with investor objectives. Constraint-aware mean-variance optimization offers a powerful framework for achieving this goal, transforming abstract risk-return trade-offs into actionable allocation strategies that perform effectively under real-world conditions. The approach underscores the importance of bridging theoretical finance and practical decision-making, ensuring that asset allocation strategies remain both efficient and feasible in complex, dynamic market environments.

VII. CONCLUSION

The findings of this study highlight the critical role of constraints in shaping realistic and practical portfolio optimization outcomes. While the classical mean-variance framework provides valuable theoretical insights into the risk–return trade-off, its assumptions often overlook the complexities inherent in actual investment environments. By incorporating realistic constraints—such as transaction costs, market regulations, and limited asset positions—the constraint-aware model achieves portfolios that are not only theoretically sound but also operationally feasible. The analysis demonstrates that constraints can improve portfolio stability, control excessive concentration, and align investment outcomes with real-world decision-making criteria. Although these constraints may slightly reduce theoretical efficiency, they significantly enhance the model’s practical applicability and robustness under market uncertainties. Overall, this study reinforces the necessity of integrating constraint-aware techniques in portfolio construction to achieve sustainable and implementable asset allocation strategies, and it opens pathways for future research in robust, multi-objective, and adaptive optimization methods for dynamic financial markets.

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