



## GRAPHS ON THE COMMUTATIVE RING $Z_n$ AND $Z_n Z_n$ IN THE NEIGHBOURHOOD: A STUDY

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### ABSTRACT

The contiguousness rundown and nearness grid portrayals can be utilized to address diagrams in PC calculations. A chart's bunching coefficient, which is a proportion of the normal thickness of its areas, likewise utilizes neighborhoods. Besides, numerous critical diagram classes can be depicted by elements of their areas or balances that connect neighborhoods together. There could be no other vertices around a separated vertex. The number of nearby vertices decides a vertex's degree. A circle associating a vertex to itself is a particular case; if such an edge exists, the vertex has a place with its own area. In this article, graphs on the commutative ring  $Z_n$  and  $Z_n Z_n$  in the neighbourhood has been discussed.

**Keywords:** Graphs, Commutative Ring,  $Z_n$ ,  $Z_n Z_n$ , Neighbourhood.

## INTRODUCTION

We set up the local diagram on the charts  $G(R)$  framed in the commutative ring  $Z_n$  and  $Z_n$  in this part, and expected some set hypothesis prerequisites on the neighborhood of vertices of  $G(R)$  to acquire the new charts we name area diagrams and meant by  $N[G]$ .  $G(R)$  produces standard, complete, and complete bipartite charts as an ensuing area diagram. In this part, we utilize the condensing nbd to represent neighborhood.

## REGULAR AND BIPARTITE GRAPHS ON $Z_n$ AND $Z_n$ NEIGHBOURHOOD GRAPHS:

The contiguousness rundown and nearness network portrayals can be utilized to address charts in PC calculations. A diagram's bunching coefficient, which is a proportion of the normal thickness of its areas, likewise utilizes neighborhoods. Moreover, numerous critical diagram classes can be portrayed by elements of their areas or balances that connect neighborhoods together.

There could be no other vertices around a secluded vertex. The number of contiguous vertices decides a vertex's degree. A circle associating a vertex to itself is a particular occurrence; if such an edge exists, the vertex has a place with its own area.

We set up the local diagram on the charts  $G(R)$  framed in the commutative ring  $Z_n$  and  $Z_n$  in this part, and expected some set hypothesis necessities on the neighborhood of vertices of  $G(R)$  to acquire the new charts we name area charts and meant by  $N[G]$ .  $G(R)$  produces normal, complete, and complete bipartite charts as a subsequent area diagram. In this section, we utilize the shortening nbd to represent neighborhood.

The nbd charts of the diagram  $G(R)$  on the commutative ring  $Z_n$  and  $Z_n$  are talked about in this segment. It has been found that the nbd charts created in this methodology are indistinguishable from the diagram  $G$ 's double chart ( $R$ ).

**Hypothesis 5.2.1:** The ring of whole numbers modulo  $n$ ,  $R=Z_n$ , fulfills every one of the standards in hypothesis 2.2.1.

$N[G]=[V(N[G],E(N[G]))]$  where  $E(N[G])=u,vG(R)/u$  and  $v$  are neighboring iff  $nbd(u) \cap nbd(v) \neq \emptyset$ . Then  $N[G]$  is the partner chart of  $G$  and is a finished bipartite diagram ( $R$ ).

Accept that the ring  $R=Z_n$  meets every one of the standards of the hypothesis 2.2.1, where  $n=2r$  ( $r \geq 2$ )

The diagram  $G(R)$  subsequently turns into a  $(2r-1-1)$ - Regular chart.  $G(R)$  characterizes the neighborhood of a vertex  $v$  as the arrangement of all vertices contiguous  $v$  in  $G(R)$  that incorporate  $v$ , and it is meant by  $nbd(v)$ . (That is,  $nbd(v)=$  The arrangement of all  $G(R)$  vertices nearby  $v$  (counting  $v$ ).

Presently we find the  $nbd$  diagram  $N[G]$ , whose vertex set is comprised of  $G(R)$  vertices and whose edge set is characterized as  $E(N[G])=u,v \in G(R) / u$  and  $v$  are nearby if  $nbd(u) \cap nbd(v) \neq \emptyset$ .

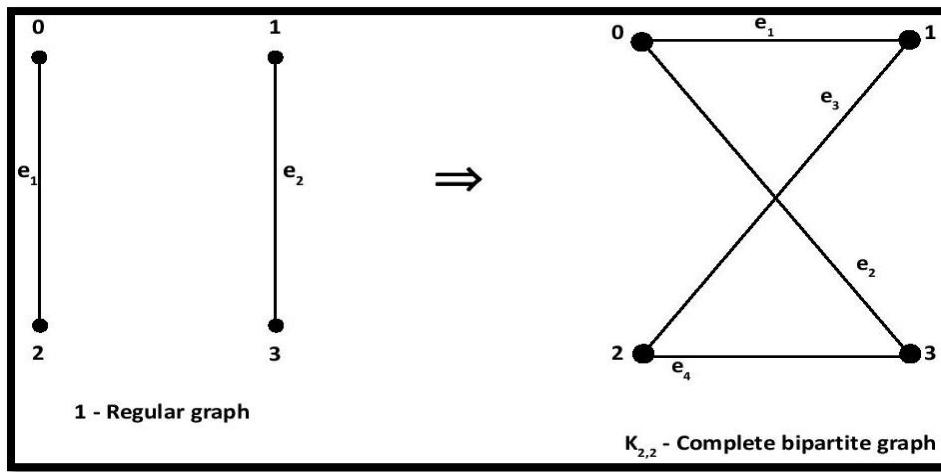
When  $n=2r$  (where  $r$  is 2),  $G(R)$  is a  $(n/2)-1$  standard chart, and  $V(G)$  rises to 4. Each vertex  $v$  in  $G(R)$  is neighboring  $((n/2)-1)$  vertices in  $G(R)$  since  $G(R)$  is a  $((n/2)-1)$  normal diagram ( $R$ ). The way that any two vertices in the  $nbd$  chart  $N[G]$  are nearby shows that the convergence of the  $nbd$ s of  $u$  and  $v$  is unfilled, for example there are no common vertices that are contiguous both  $u$  and  $v$ . Each vertex in  $G(R)$  is nearby  $((n/2)-1)$  and not to  $n/2$  vertices since it is a  $((n/2)-1$  – customary diagram. (Every vertex degree in a straightforward organization with  $n$  vertices is  $n-1$ ) therefore, every vertex  $v$  in  $N[G]$  is nearby those  $G(R)$  vertices that are not adjoining to  $v$ . The subsequent chart  $N[G]$  is then a  $n/2$ -normal diagram.  $N[G]$  is likewise a total bipartite diagram with  $Kn/2, n/2$  hubs. Since adjoining vertices in the diagram  $G(R)$  are not contiguous in the chart  $N[G]$ , as well as the other way around. Accordingly,  $N[G]$  is a double diagram of  $G$ . ( $R$ ).

**Outline 5.2.2:** Let  $R=Z_n$  be a commutative ring of integers modulo  $n$ , where  $n=2r$  ( $r \geq 2$ )

**Case (i):** Let  $r=2$  and  $n=22$ ,  $|R|=4$ , i.e.,  $R=\{0,1,2,3\}$ . Then the arrangement of all nilpotent components  $S=\{2\}$  and  $G(R)$  is a 1-ordinary diagram with  $V(G(R))=\{0,1,2,3\}$ . Presently  $nbd[0]=\{0,2\}$   $nbd[1]=\{1,3\}$

$=\{1,3\}$   $nbd[2]=\{0,2\}$   $nbd[3]=\{1,3\}$ . Since  $nbd[0] \cap nbd[1]=\emptyset$ ,  $nbd[0] \cap nbd[3]=\emptyset$ ,  $nbd[1] \cap nbd[2]=\emptyset$ ,  $nbd[2] \cap nbd[3]=\emptyset$ . Subsequently 0 and 1, 0 and 3, 1 and 2, 2 and 3 are contiguous in  $N[G]$ . Subsequently  $E(N[G])=\{e_1, e_2, e_3, e_4\}$  where  $e_1=(0,1)$   $e_2=(0,3)$   $e_3=(1,2)$   $e_4=(2,3)$ .

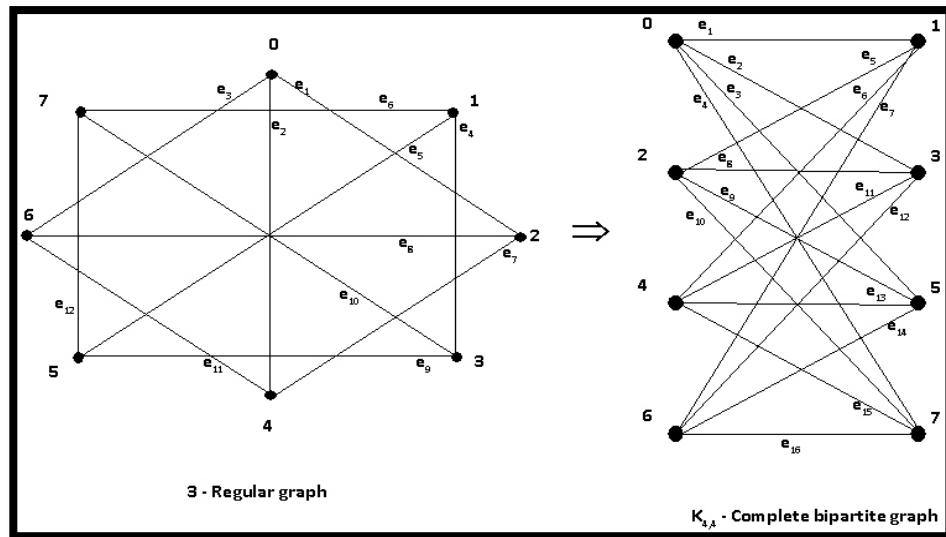
Subsequently the nbd diagram  $N[G]$  of  $G(R)$  is a  $K_{2,2}$ -Complete bipartite chart.



**Case(ii):** Let  $r=3$  and  $n=2^3$ ,  $|R|=8$  i.e.,  $R=\{0,1,2,3,4,5,6,7\}$  the set of all nilpotent element  $S=\{2,4,6\}$  then  $G(R)$  is a 3-regular graph with  $V(G(R))=\{0,1,2,3,4,5,6,7\}$  Now  $nbd[0]=\{0,2,4,6\}$   $nbd[1]=\{1,3,5,7\}$   $nbd[2]=\{0,2,4,6\}$   $nbd[3]=\{1,3,5,7\}$ ,  $nbd[4]=\{0,2,4,6\}$   $nbd[5]=\{1,3,5,7\}$   $nbd[6]=\{0,2,4,6\}$   $nbd[7]=\{1,3,5,7\}$ . Since  $nbd[0] \cap nbd[1]=\phi$ ,  $nbd[0] \cap nbd[3]=\phi$ ,  $nbd[0] \cap nbd[5]=\phi$ ,  $nbd[0] \cap nbd[7]=\phi$ ,  $nbd[2] \cap nbd[1]=\phi$ ,  $nbd[2] \cap nbd[3]=\phi$ ,  $nbd[2] \cap nbd[5]=\phi$ ,  $nbd[2] \cap nbd[7]=\phi$ ,  $nbd[4] \cap nbd[1]=\phi$ ,  $nbd[4] \cap nbd[3]=\phi$ ,  $nbd[4] \cap nbd[5]=\phi$ ,  $nbd[0] \cap nbd[7]=\phi$ ,  $nbd[6] \cap nbd[1]=\phi$ ,  $nbd[6] \cap nbd[3]=\phi$ ,  $nbd[6] \cap nbd[5]=\phi$ ,  $nbd[6] \cap nbd[7]=\phi$ . Therefore 0 and 2, 0 and 4 etc. are adjacent in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$  where  $e_1=(0,1)$   $e_2=(0,3)$   $e_3=(0,5)$   $e_4=(0,7)$   $e_5=(2,1)$   $e_6=(2,3)$   $e_7=(2,5)$   $e_8=(2,7)$   $e_9=(4,1)$   $e_{10}=(4,3)$   $e_{11}=(4,5)$   $e_{12}=(4,7)$   $e_{13}=(1,6)$   $e_{14}=(3,6)$   $e_{15}=(5,6)$   $e_{16}=(6,7)$ .

Thus,  $G(R_{nbd})$ 's diagram  $N[G]$  is a  $K_{4,4}$ -Complete bipartite chart.

**End:** If  $n=2r$  ( $r \geq 2$ ), the nbd chart  $N[G]$  of  $G(R)$  is a  $K_{n/2, n/2}$ -Complete bipartite diagram, as displayed in the graph above.



The relationship between  $n$  and  $N[G]$  is seen in the table below.

| $r$ | Number of vertices $n=2^r$ ( $r \geq 2$ ) | Graph $N[G]=K_{n/2, n/2}$ -Complete bipartite Graph |
|-----|---|---|
| 2   | $2^2$                                     | $K_{2,2}$ - Complete bipartite graph.               |
| 3   | $2^3$                                     | $K_{4,4}$ - Complete bipartite graph.               |
| .   | .   | .   |
| .   | .   | .   |

**Hypothesis 5.2.3:** Let  $R=Z_n$  be the ring of numbers modulo  $n$ , fulfills every one of the states of hypothesis 2.2.3. Define  $N[G]=[V(N[G], E(N[G]))]$  be the local diagram of  $G(R)$ ,  $V(N[G])$  is the vertex set and  $E(N[G])$  is the edge set of  $N[G]$  where  $E(N[G])=\{u, v \in G(R) : u \text{ and } v \text{ are neighboring iff } \text{nbd}(u) \cap \text{nbd}(v) = \emptyset\}$ . Then  $N[G]$  is a **Regular chart** and it is additionally the double diagram of  $G(R)$ .

Expect that the ring  $R=Z_n$  fulfills all of the hypothesis 2.2.3's rules. The chart  $G(R)$  is a

bipartite diagram in case  $n$  is a few.  $G(R)$  characterizes the neighborhood of a vertex  $v$  as the arrangement of all vertices adjoining  $v$  in  $G(R)$  that incorporate  $v$ , and it is meant by  $nbd(v)$ . (That is,  $nbd(v)$ = The arrangement of all  $G(R)$  vertices nearby  $v$  (counting  $v$ )

The  $nbd$  diagram  $N[G]$  is presently found.  $N[G]$ 's vertices are indistinguishable from  $G$ 's vertices ( $R$ ).  $N(G)$ 's set is characterized as  $E(N[G])=u,vG(R)/u$  and  $v$  are close by if  $nbd(u)$   $nbd(v) =$  If  $n$  is even ( $n \geq 2$ ), the diagram  $G(R)$  is a bipartite chart  $(n/2, n/2)$ .

Essentially, in case  $n$  is odd ( $n \geq 3$ ), the diagram  $G(R)$  is a bipartite chart  $((n-1)/2, (n-1)/2)$ . Assuming  $n$  is even and ( $n \geq 4$ ) ( $n$  is odd ( $n \geq 5$ )), the  $nbd$  diagram  $N[G]$  is characterized. The quantity of vertices in  $N[G]$  rises to the quantity of vertices in  $G(R)$ , and each vertex in  $N[G]$  is adjoining those  $G(R)$  vertices that are not neighboring  $v$  in  $G(R)$ . Since  $G(R)$  is a  $(n/2, n/2)$  bipartite diagram (or  $(n-1)/2, (n-1)/2$ ) bipartite chart, the  $nbd$  diagram  $N[G]$  of  $G(R)$  is a  $(n-2)$  or  $(n-3)$  customary diagram. Moreover, the quantity of adjoining vertices in  $G(R)$  isn't adjoining in  $N[G]$ , as well as the other way around. Subsequently,  $N[G]$  is a double diagram of  $G(R)$ .

On the off chance that we proceed thusly whether or not  $n$  is odd or even, we have the  $nbd$  chart.  $G(R)$ 's  $[G]$  is a

**This is a standard graph.**

Let  $R=Z_n$  be a commutative ring of integers modulo  $n$ , where  $n$  is either 4 or 5.

**Case(i):** Let  $n=4$  then  $R= \{0,1,2,3\}$ . So the graph is a  $(2,2)$ -bipartite graph with  $V(G(R))=\{0,1,2,3\}$ . Now  $nbd[0]=\{0,1\}$   $nbd[1]=\{0,1\}$   $nbd[2]=\{2,3\}$   $nbd[3]=\{2,3\}$ .

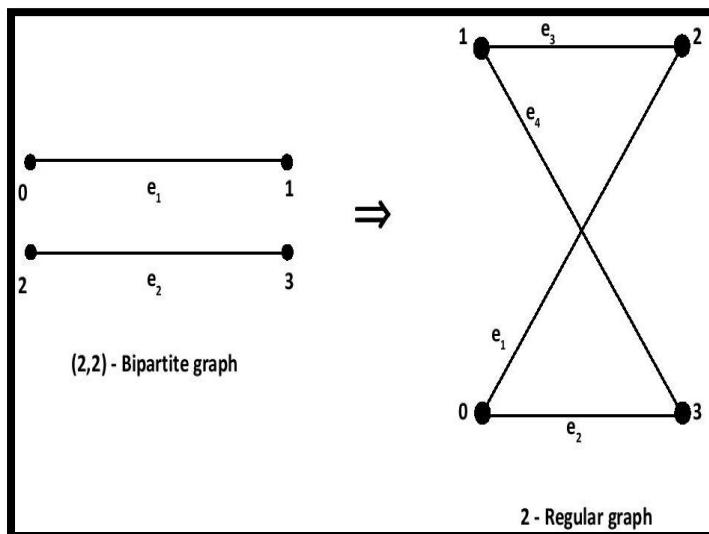
Since  $nbd[0] \cap nbd[2] = \emptyset$ ,  $nbd[0] \cap nbd[3] = \emptyset$ ,  $nbd[1] \cap nbd[2] = \emptyset$ ,  $nbd[1] \cap nbd[3] = \emptyset$ .

$[3]=\emptyset$ ,  $nbd[2] \cap nbd[3] = \emptyset$ . Therefore 0 and 3, 1 and 3 etc. are adjacent in  $N[G]$ , Hence  $E(N[G])=\{e_1, e_2, e_3, e_4\}$  where  $e_1=(0,2)$   $e_2=(0,3)$   $e_3=(1,2)$   $e_4=(1,3)$ .

Hence the nbd chart  $N[G]$  of  $G(R)$  is a 2-Regular diagram.

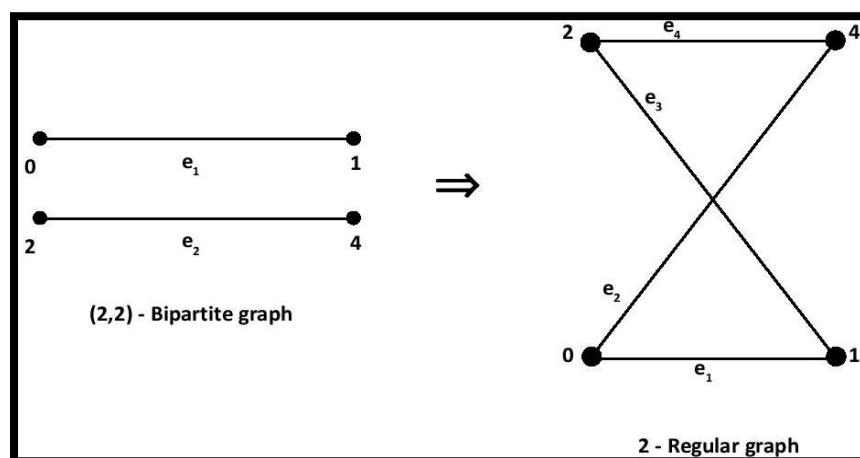
Essentially Let  $n=5$  then  $|R|=4$ , i.e.,  $R=\{0,1,2,3,4\}$ . So the chart is a (2,2)- bipartite diagram with  $V(G(R))=\{0,1,2,4\}$ . Now  $nbd[0]=\{0,1\}$   $nbd[1]=\{0,1\}$   $nbd[2]=\{2,4\}$   $nbd[4]=\{2,4\}$ . Since  $nbd[1] \cap nbd[2] = \emptyset$ ,  $nbd[0] \cap nbd[2] = \emptyset$ ,  $nbd[0] \cap nbd[4] = \emptyset$ ,  $nbd[1] \cap nbd[2] = \emptyset$ ,  $nbd[1] \cap nbd[4] = \emptyset$ ,  $nbd[2] \cap nbd[4] = \emptyset$ . Hence 0 and 1, 0 and 4 are neighboring in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4\}$  where  $e_1=(0,1)$   $e_2=(0,4)$   $e_3=(1,2)$   $e_4=(2,4)$ .

In this manner the nbd diagram  $N[G]$  of  $G(R)$  is a 2-Regular chart

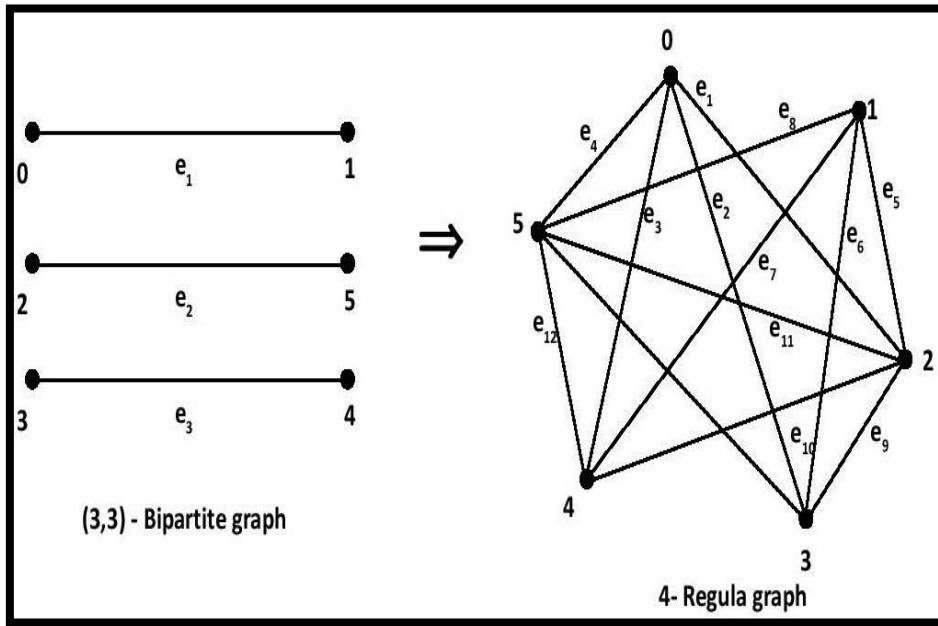


**Case(ii):** Let  $n=6$  then  $|R|=6$  i.e.,  $R=\{0,1,2,3,4,5\}$ . So the chart  $G(R)$  is a (3,3)- bipartite diagram with  $V(G(R))=\{0,1,2,3,4,5\}$ . Now  $nbd[0]=\{0,1\}$   $nbd[1]=\{0,1\}$   $nbd[2]=\{2,5\}$

$nbd[3]=\{3,4\}$   $nbd[4]=\{3,4\}$   $nbd[5]=\{2,5\}$ . Since  $nbd[0] \cap nbd[2] = \emptyset$ ,  $nbd[0] \cap nbd[5] = \emptyset$



$[3] = \emptyset$ ,  $\text{nbd}[0] \cap \text{nbd}[4] = \emptyset$ ,  $\text{nbd}[0] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[2] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[3] = \emptyset$



$[3] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[4] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[5] = \emptyset$   $\text{nbd}[2] \cap \text{nbd}[3] = \emptyset$ ,  $\text{nbd}[2] \cap$

$\text{nbd}[4] = \emptyset$ ,  $\text{nbd}[2] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[3] \cap \text{nbd}[4] = \emptyset$ ,  $\text{nbd}[3] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[4]$

$\cap \text{nbd}[5] = \emptyset$ . Hence 0 and 2, 0 and 4 and so on are nearby in  $N[G]$ . Hence  $E(N[G]) = \{e1, e2, e3, e4, e5, e6, e7, e1, e8, e9, e10, e11, e12\}$  where  $e1 = (0, 2)$   $e2 = (0, 3)$   $e3 = (0, 4)$   $e4 = (0, 5)$   $e5 = (1, 2)$   $e6 = (1, 3)$   $e7 = (1, 4)$   $e8 = (1, 5)$   $e9 = (2, 3)$   $e10 = (2, 4)$   $e11 = (3, 5)$   $e12 = (4, 5)$ .

Hence the nbd chart  $N[G]$  of  $G(R)$  is a 4-Regular diagram.

Likewise Let  $n=7$  then  $|R| = 7 = \{0, 1, 2, 3, 4, 5, 6\}$ . So the chart  $G(R)$  is a (2,2)- bipartite diagram with  $V(G(R)) = \{0, 1, 2, 3, 5, 6\}$ . Now  $\text{nbd}[0] = \{0, 1\}$   $\text{nbd}[1] = \{0, 1\}$   $\text{nbd}[2] = \{2, 6\}$

$\text{nbd}[3] = \{3, 5\}$   $\text{nbd}[5] = \{3, 5\}$   $\text{nbd}[6] = \{2, 6\}$ . Since  $\text{nbd}[0] \cap \text{nbd}[2] = \emptyset$ ,  $\text{nbd}[0] \cap \text{nbd}$

$[3] = \emptyset$ ,  $\text{nbd}[0] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[0] \cap \text{nbd}[6] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[2] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}$

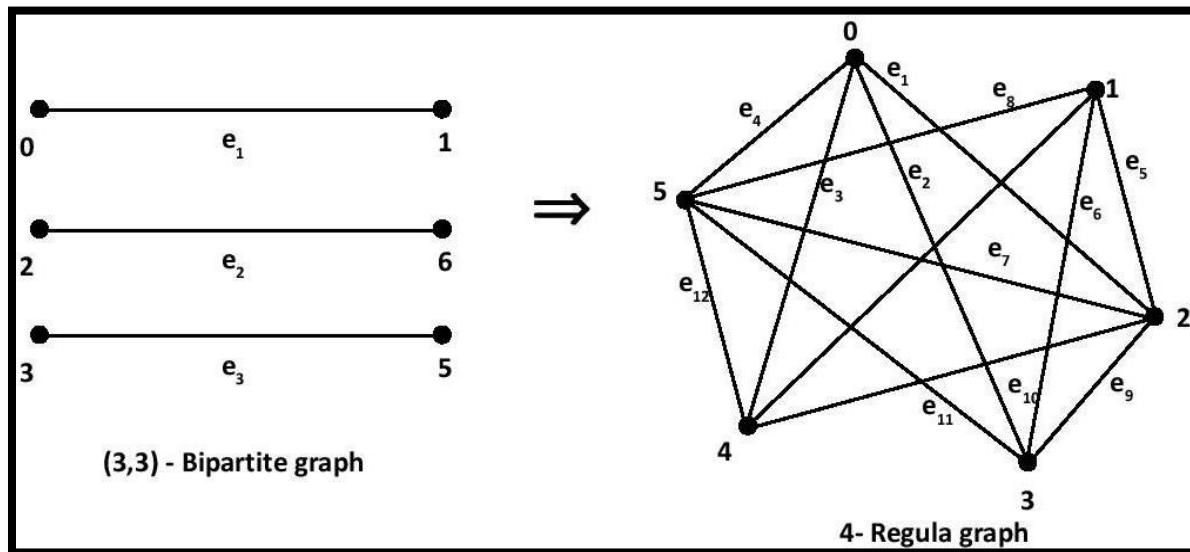
$[3] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[1] \cap \text{nbd}[6] = \emptyset$ ,  $\text{nbd}[2] \cap \text{nbd}[3] = \emptyset$ ,  $\text{nbd}[2] \cap \text{nbd}$

$[5] = \emptyset$ ,  $\text{nbd}[2] \cap \text{nbd}[6] = \emptyset$ ,  $\text{nbd}[3] \cap \text{nbd}[5] = \emptyset$ ,  $\text{nbd}[3] \cap \text{nbd}[6] = \emptyset$ ,  $\text{nbd}[5] \cap \text{nbd}$

[6] = $\phi$ . Accordingly 0 and 2, 0 and 4 and so forth are adjoining in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4,$

$e_5, e_6, e_7, e_1, e_8, e_9, e_{10}, e_{11}, e_{12}\}$  where  $e_1 = (0,2)$   $e_2 = (0,3)$   $e_3 = (0,5)$   $e_4 = (0,6)$   $e_5 = (1,2)$   $e_6 = (1,3)$

$e_7 = (1,5)$   $e_8 = (1,6)$   $e_9 = (2,3)$   $e_{10} = (2,5)$   $e_{11} = (3,6)$   $e_{12} = (5,6)$ .



Subsequently, the  $N[G]$  of  $G(R)$  nbd chart is a 4-Regular diagram.

**End:** If  $n$  is even or odd, the nbd chart  $N[G]$  of  $G(R)$  is a Regular diagram, as displayed in the outline above

**The relationship between  $n$  and  $N[G]$  is seen in the table below**

| Number of vertices<br>$n \geq 4$ | $N[G] = (n-2)$ - Regular<br>graph | $N[G] = (n-3)$ - Regular<br>graph |
|----------------------------------|-----------------------------------|-----------------------------------|
|                                  | $n$ is even ( $n \geq 4$ )        | $n$ is odd ( $n \geq 5$ )         |
| 4                                | 2-Regular graph                   |                                   |
| 5                                |                                   | 2-Regular graph                   |
| 6                                | 4-Regular graph                   |                                   |
| 7                                | .                                 | 4-Regular graph                   |
| .                                | .                                 | .                                 |
| .                                | .                                 | .                                 |
| .                                | .                                 | .                                 |

Leave  $R = \mathbb{Z}_n$  alone the ring of whole numbers modulo  $n$ , which meets the hypothesis' models in general. 2.2.5. Characterize  $N[G] = [V(N[G], E(N[G]))]$  as the local diagram of  $G(R)$ , where  $V(N[G])$  is the vertex set and  $E(N[G])$  is the edge set, and  $E(N[G]) = u, v \in G(R) / u$  and  $v$  are neighboring iff  $nbd(u) \cap nbd(v) \neq \emptyset$ . Then  $N[G]$  is a Regular chart, just as  $G$ 's double chart ( $R$ ).

Accept that the ring  $R = \mathbb{Z}_n$  fulfills all of the hypothesis 2.2.5's models. At the point when  $n = 4$  or  $5$ , the chart  $G(R)$  is a bipartite diagram.  $G(R)$  characterizes the neighborhood of a vertex  $v$  as the arrangement of all vertices adjoining  $v$  in  $G(R)$  that incorporate  $v$ , and it is signified by  $nbd(v)$ . (That is,  $nbd(v) = \{u \in G(R) \mid u \text{ is contiguous to } v\}$  counting  $v$ )

Presently we find the  $nbd$  chart  $N[G]$ , whose vertex set is comprised of  $G(R)$  vertices and whose edge set is characterized as  $E(N[G]) = u, v \in G(R) / u$  and  $v$  are contiguous if  $nbd(u) \cap nbd(v) \neq \emptyset$ .

In the event that  $n$  is even ( $n \neq 4$ )  $G(R)$  is a bipartite diagram  $((n-2)/2, (n-2)/2)$ . Additionally, in case  $n$  is odd ( $n \neq 5$ ), the diagram  $G(R)$  is a bipartite chart  $((n-3)/2, (n-3)/2)$ . Assuming  $n$  is even and ( $n \neq 6$ ) ( $n$  is odd ( $n \neq 7$ )), the quantity of vertices of  $N[G]$  is the quantity of vertices of  $G(R)$ , and each vertex of  $N[G]$  is neighboring those vertices of  $G(R)$  that are not contiguous  $v$  in  $G(R)$ . Since  $G(R)$  is a  $((n-2)/2, (n-2)/2)$  bipartite chart, and each vertex in one segment is nearby precisely one vertex in the other, each vertex in  $N[G]$  is contiguous ( $n-4$ ) or ( $n-5$ ) vertices. Accordingly, the  $nbd$  chart  $N[G]$  of  $G(R)$  is either a  $(n-4)$ - normal diagram or a  $(n-5)$ - ordinary diagram. Furthermore, the quantity of nearby vertices in  $G(R)$  isn't adjoining in  $N[G]$ , as well as the other way around. Accordingly,  $N[G]$  is a double chart of  $G(R)$ .

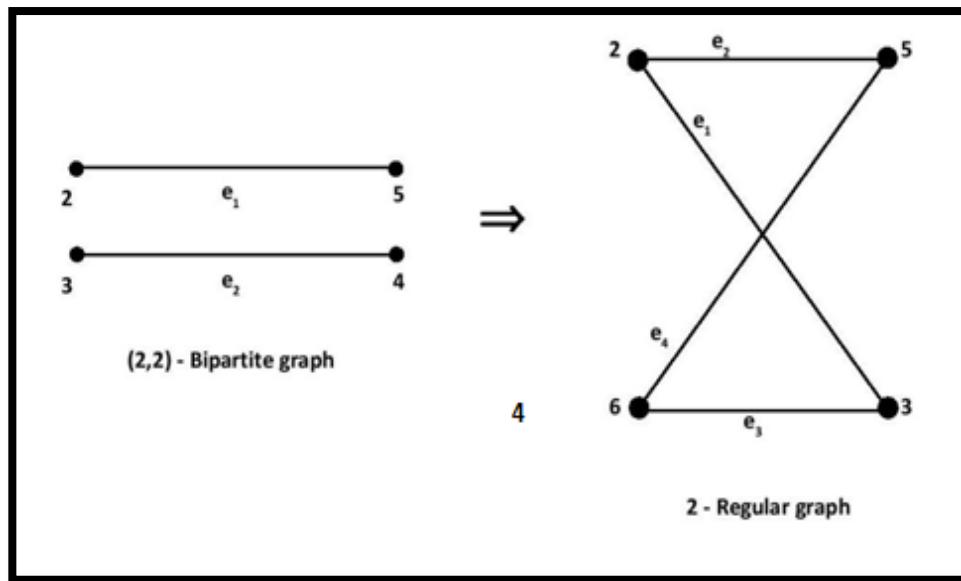
Assuming  $n$  is odd or even, the  $nbd$  diagram  $N[G]$  of  $G(R)$  is a

### Regular graph

Leave  $R = \mathbb{Z}_n$  alone a commutative ring of whole numbers modulo  $n$ , where  $n$  is either  $6$  or  $7$ .

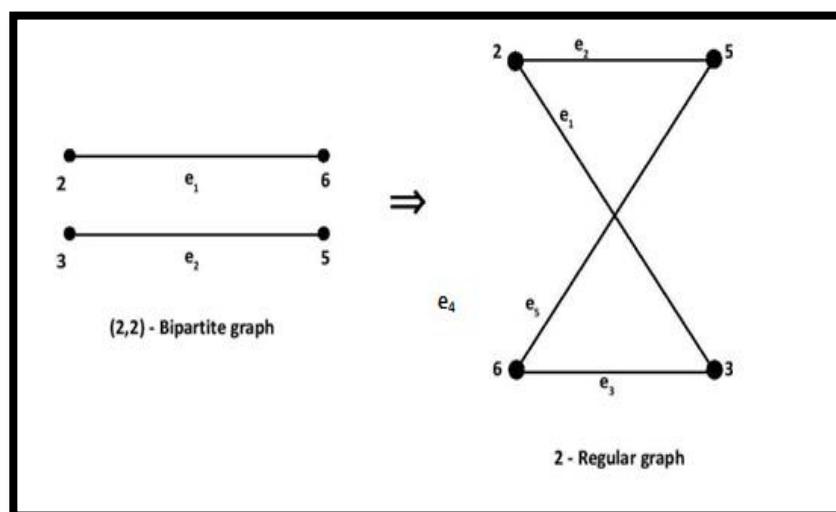
**Case(i):** Assume  $n=6$  and  $R=0,1,2,3,4,5$ .

Therefore, the chart is a bipartite  $(2,2)$  diagram with  $V(G(R)) = 2, 3, 4, 5$ .



Thusly the nbd diagram  $N[G]$  of  $G(R)$  is a 2-customary chart.

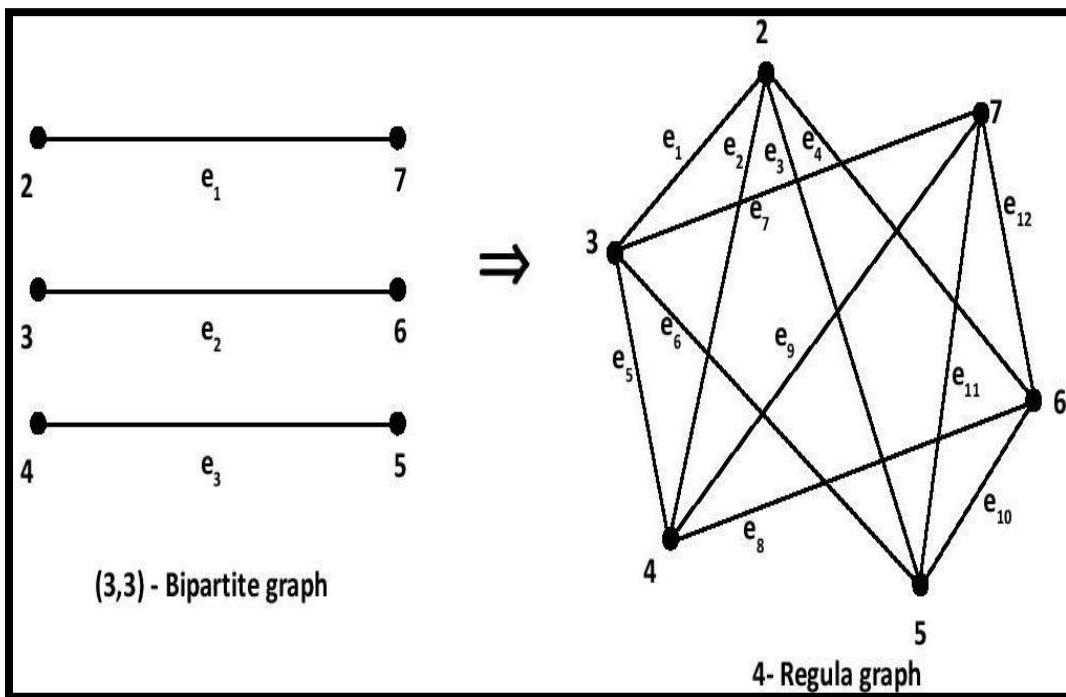
Also Let  $n=7$  then  $R= \{0,1,2,3,4,5,6\}$ . So the diagram is a (1,1)- bipartite chart with  $V(G(R))=\{2,3,5,6\}$ . Now  $nbd[2]=\{2,6\}$        $nbd[3]=\{3,5\}$   $nbd[5]=\{3,5\}$   $nbd[6]=\{2,6\}$ . Since  $nbd[2] \cap nbd[3] = \emptyset$ ,  $nbd[2] \cap nbd[5] = \emptyset$ ,  $nbd[3] \cap nbd[6] = \emptyset$ ,  $nbd[5] \cap nbd[6] = \emptyset$ . Therefore 2 and 3, 2 and 5 and so forth are contiguous in  $N[G]$ . Hence  $E(N[G])=\{e1, e2, e3, e4\}$  where  $e1=(2,3)$   $e2=(2,5)$   $e3=(6,3)$   $e4=(6,5)$ .



Thusly the nbd chart  $N[G]$  of  $G(R)$  is a 2-ordinary diagram

Case(ii): Let  $n=8$  then  $R=\{0,1,2,3,4,5,6,7\}$ . So the diagram  $G(R)$  is a  $(3,3)$ - bipartite chart with,  $V(G(R))=\{2,3,4,5,6,7\}$ . Presently  $nbd[2]=\{2,7\}$   $nbd[3]=\{3,6\}$   $nbd[4]=\{4,5\}$   $nbd[5]=\{5,7\}$   $nbd[6]=\{6,2\}$   $nbd[7]=\{7,3\}$ . Since  $nbd[2] \cap nbd[3]=\emptyset$ ,  $nbd[2] \cap nbd[4]=\emptyset$ ,  $nbd[2] \cap nbd[5]=\emptyset$ ,  $nbd[2] \cap nbd[6]=\emptyset$ ,  $nbd[3] \cap nbd[4]=\emptyset$ ,  $nbd[3] \cap nbd[5]=\emptyset$ ,  $nbd[3] \cap nbd[7]=\emptyset$ ,  $nbd[4] \cap nbd[6]=\emptyset$ ,  $nbd[4] \cap nbd[7]=\emptyset$ ,  $nbd[5] \cap nbd[6]=\emptyset$ ,  $nbd[5] \cap nbd[7]=\emptyset$ ,  $nbd[6] \cap nbd[7]=\emptyset$ . Thusly 2 and 5, 2 and 6 and so on are neighboring in  $N[G]$ . Thus  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$  where  $e_1=(2,3)$   $e_2=(2,4)$

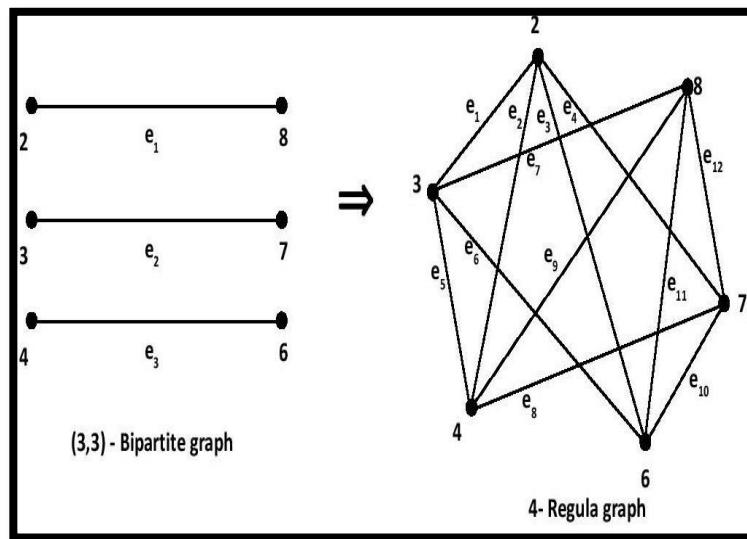
$e_3=(2,5)$   $e_4=(2,6)$   $e_5=(3,4)$   $e_6=(3,5)$   $e_7=(3,7)$   $e_8=(4,6)$   $e_9=(4,7)$   $e_{10}=(5,6)$   $e_{11}=(5,7)$   $e_{12}=(6,7)$ .



Thusly the nbd chart  $N[G]$  of  $G(R)$  is a 4-ordinary diagram.

Also Let  $n=9$  then  $R=\{0,1,2,3,4,5,6\}$ . So the chart  $G(R)$  is a  $(3,3)$ - bipartite diagram with  $V(G(R))=\{2,3,4,6,7,8\}$ . Now  $nbd[2]=\{2,8\}$   $nbd[3]=\{3,7\}$   $nbd[4]=\{4,6\}$   $nbd[6]=\{4,6\}$   $nbd[7]=\{3,7\}$   $nbd[8]=\{2,8\}$ . Since  $nbd[2] \cap nbd[3]=\emptyset$ ,  $nbd[2] \cap nbd[4]=\emptyset$ ,  $nbd[2] \cap nbd[6]=\emptyset$ ,  $nbd[2] \cap nbd[7]=\emptyset$ ,  $nbd[3] \cap nbd[4]=\emptyset$ ,  $nbd[3] \cap nbd[6]=\emptyset$ ,  $nbd[3] \cap nbd[7]=\emptyset$ ,  $nbd[4] \cap nbd[6]=\emptyset$ ,  $nbd[4] \cap nbd[7]=\emptyset$ ,  $nbd[6] \cap nbd[7]=\emptyset$ .

$\text{nbd}[8] = \emptyset$ ,  $\text{nbd}[4] \cap \text{nbd}[7] = \emptyset$ ,  $\text{nbd}[4] \cap \text{nbd}[8] = \emptyset$ ,  $\text{nbd}[6] \cap \text{nbd}[7] = \emptyset$ ,  $\text{nbd}[6] \cap \text{nbd}[8] = \emptyset$ ,  $\text{nbd}[6] \cap \text{nbd}[8] = \emptyset$  and  $\text{nbd}[7] \cap \text{nbd}[8] = \emptyset$ . Therefore 2 and 6, 2 and 7 and so forth are neighboring in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_1, e_8, e_9, e_{10}, e_{11}, e_{12}\}$  where  $e_1 = (2,3)$   $e_2 = (2,4)$   $e_3 = (2,6)$   $e_4 = (2,7)$   $e_5 = (3,4)$   $e_6 = (3,6)$   $e_7 = (3,8)$   $e_8 = (4,7)$   $e_9 = (4,8)$   $e_{10} = (6,7)$   $e_{11} = (6,8)$   $e_{12} = (7,8)$ .



Accordingly, the  $N[G]$  of  $G(R)$  nbd chart is a 4-ordinary diagram.

**End:** If  $n$  is even or odd, the nbd chart  $N[G]$  of  $G(R)$  is an ordinary chart, as displayed in the outline above.

**The connection among  $n$  and  $N[G]$  is found in the table underneath.**

| Number of vertices<br>$n \geq 6$ | $N[G] = (n-4)$ Regular graph | $N[G] = (n-5)$ Regular graph |
|----------------------------------|------------------------------|------------------------------|
|                                  | $n$ is even ( $n \geq 6$ )   | $n$ is odd ( $n \geq 7$ )    |
| 6                                | 2-Regular graph              |                              |
| 7                                |                              | 2-Regular graph              |
| 8                                | 4-Regular graph              |                              |
| 9                                | .                            | 4-Regular graph              |
| .                                | .                            | .                            |
| .                                | .                            | .                            |
| .                                | .                            | .                            |

**Hypothesis 5.2.7:** The ring of whole numbers modulo  $n$ ,  $R=Z_n$ , fulfills each of the measures in hypothesis 2.2.7.  $N[G]=[V(N[G],E(N[G])]$  where  $E(N[G])=u,vG(R)/u$  and  $v$  are nearby iff  $nbd(u) nbd(v) = N[G]$  is then a Regular diagram, just as the double chart  $G$ . (R).

**Confirmation:** Assume that the ring  $R= Z_n$  fulfills all of the hypothesis 2.2.7's standards. At the point when  $n = 3$  or  $4$ , the diagram  $G(R)$  is a bipartite chart.  $G(R)$  characterizes the neighborhood of a vertex  $v$  as the assortment of those vertices near  $v$  in  $G(R)$  that incorporate  $v$ .

$nbd nbd nbd nb (v)$ . (That is,  $nbd(v)=$  The arrangement of all  $G(R)$  vertices contiguous  $v$  (counting  $v$ )

Presently we find the  $nbd$  diagram  $N[G]$ , whose vertex set is comprised of  $G(R)$  vertices and whose edge set is characterized as  $E(N[G])=u,vG(R)/u$  and  $v$  are adjoining if  $nbd(u) nbd(v) =$ .

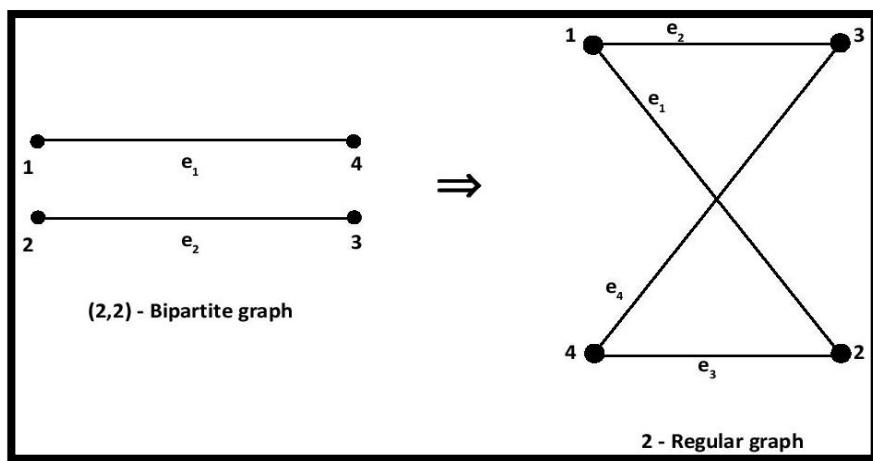
Assuming  $n$  is even ( $n \neq 4$ )  $G(R)$  is a bipartite chart  $((n-2)/2,(n-2)/2)$ . Also, in case  $n$  is odd ( $n \neq 3$ ), the chart  $G(R)$  is a bipartite diagram  $((n-1)/2,(n-1)/2)$ . Assuming  $n$  is even and ( $n \neq 6$ ) ( $n$  is odd ( $n \neq 5$ )), the  $nbd$  chart  $N[G]$  is characterized. The quantity of vertices in  $N[G]$  rises to the quantity of vertices in  $G(R)$ , and each vertex in  $N[G]$  is nearby those  $G(R)$  vertices that are not adjoining  $v$  in  $G$ . (R). Since  $G(R)$  is a  $((n-2)/2,(n-2)/2)$  bipartite diagram or  $((n-1)/2,(n-1)/2)$  bipartite chart, the  $nbd$  chart  $N[G]$  of  $G(R)$  is a  $(n-4)$  or  $(n-3)$  customary chart, individually. Likewise, the quantity of nearby vertices in  $G(R)$  isn't adjoining in  $N[G]$ , as well as the other way around. Thus,  $N[G]$  is a double chart of  $G$ . (R).

Proceeding as such, we get the  $nbd$  diagram  $N [G]$  of  $G(R)$  in case  $n$  is odd or even.

### This is a standard graph.

Leave  $R=Z_n$  alone a commutative ring of numbers modulo  $n$ , where  $n$  is either 5 or 6.

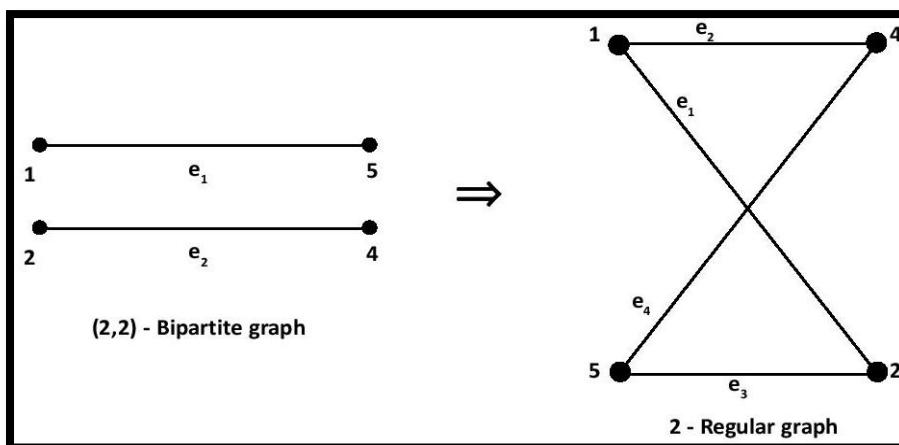
Case(i): Assume  $n=5$  and  $R=0,1,2,3,4$ . Accordingly, the diagram is a bipartite  $(2,2)$  chart with  $V(G(R))=1,2,3,4$ .  $nbd[1]$  now approaches  $1,4 = 2,3$   $nbd[2] = 2,3$   $nbd[3] = 2,3$   $nbd[4] = 1,4$ . Since  $nbd[1] nbd[2] =$ ,  $nbd[1] nbd[3] =$ ,  $nbd[2] nbd[4] =$ ,  $nbd[3] nbd[4] =$ ,  $nbd[3] nbd[4] =$  accordingly, in  $N[G]$ , 1 and 2,4 and 3, etc are contiguous. Accordingly,  $E(N[G])=e1, e2, e3, e4$ , with  $e1=(1,2), e2=(1,3), e3=(2,4)$ , and  $e4=(3,4)$ .



Thus, the  $N[G]$  of  $G(R)$  nbd diagram is a 2-Regular chart.

Also Let's say  $n=6$  and  $R=0,1,2,3,4,5$ . Thus, the diagram is a bipartite (2,2) chart, with  $V(G(R))=2,3,5,6$ .  $nbd[1] = 1,5$  now 2,4  $nbd[2] = 2,4$   $nbd[4] = 1,5$   $nbd[5] = 1,5$   $nbd[5] = 1,5$   $nbd[5]$  nb Since  $nbd[1] = nbd[2] =$ ,  $nbd[1] = nbd[4] =$ ,  $nbd[2] = nbd[4] =$ ,  $nbd[2] = nbd[5] =$ , thus, in  $N[G]$ , 1 and 4,5 and 4, etc are contiguous. Subsequently,  $E(N[G]) = e_1, e_2, e_3, e_4$  with  $e_1 = (1,2)$ ,  $e_2 = (1,4)$ ,  $e_3 = (2,5)$ , and  $e_4 = (4,5)$

Subsequently, the  $N[G]$  of  $G(R)$  nbd chart is a 2-Regular diagram.



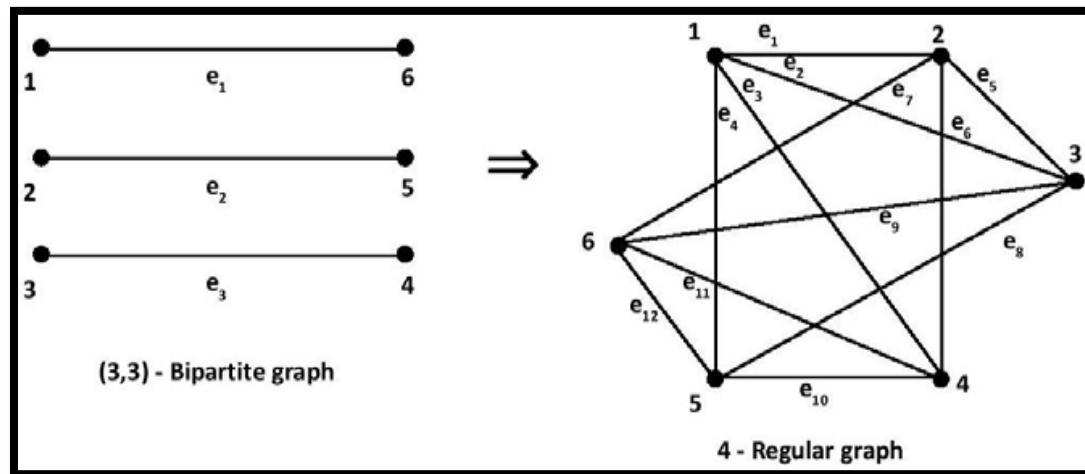
**Case(ii):** Assume  $n=7$ , and  $R=0,1,2,3,4,5,6$ .

Subsequently, the chart has  $V(G(R))=1,2,3,4,5,6$  and is a (3,3)- Bipartite diagram.

Presently,  $nbd[1] = 1,6$ , and  $nbd[2] = 2,5$ .  $nbd[3] = 3,4$   $nbd[3] = 3,4$   $nbd[3] = 3,4$   $nbd[4] = 3,4$   $nbd[4] = 3,4$   $nbd[4] = 3,4$   $nbd[4] = 2,5$   $nbd[5] = 2,5$   $nbd[5] = 2,5$   $nbd[5] = 1,6$   $nbd[6] = 1,6$   $nbd[6] = 1,6$  n Since  $nbd[1] = nbd[2] =$ ,  $nbd[1] = nbd[3] =$ ,  $nbd[1] = nbd[4] =$ ,  $nbd[1] = nbd[5] =$ ,  $nbd$

[2] nbd [3] =, nbd [2] nbd [4] =, nbd [2] nb subsequently, in  $N[G]$ , 1 and 2, 1 and 3, etc are neighboring. Subsequently,  $E(N[G]) = e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}$  with  $e_1 = (1,2)$

$e_2 = (1,3), e_3 = (1,4), e_4 = (1,5), e_5 = (2,3), e_6 = (2,4), e_7 = (2,6), e_8 = (3,5), e_9 = (3,6)$ , and  $e_{10} = (3,7)$   $(4,5) e_{12} = (4,6) e_{11} = (4,6) (5,6)$ .



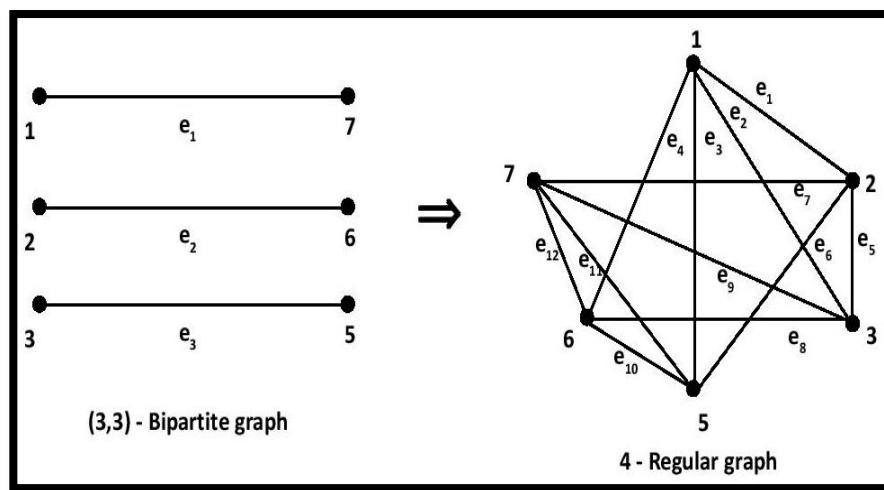
Hence the nbd chart  $N[G]$  of  $G(R)$  is a 4-Regular diagram.

Essentially Let  $n=8$  then  $R= \{0,1,2,3,4,5,6,7\}$ . So the chart is a (3,3)- Bipartite diagram with  $V(G(R)) = \{1,2,3,4,5,6\}$ . Now  $nbd[1] = \{1,7\}$   $nbd[2] = \{2,6\}$   $nbd[3] = \{3,5\}$   $nbd[5] = \{3,5\}$

$nbd[6] = \{2,6\}$   $nbd[7] = \{1,7\}$ . Since  $nbd[1] \cap nbd[2] = \emptyset$ ,  $nbd[1] \cap nbd[3] = \emptyset$ ,  $nbd[1]$

$\cap nbd[5] = \emptyset$ ,  $nbd[1] \cap nbd[6] = \emptyset$ ,  $nbd[2] \cap nbd[3] = \emptyset$ ,  $nbd[2] \cap nbd[5] = \emptyset$ ,  $nbd$

$[2] \cap nbd[7] = \emptyset$ ,  $nbd[3] \cap nbd[6] = \emptyset$ ,  $nbd[3] \cap nbd[7] = \emptyset$ ,  $nbd[5] \cap nbd[6] = \emptyset$ ,  $nbd[5] \cap nbd[7] = \emptyset$ ,  $nbd[6] \cap nbd[7] = \emptyset$ . In this manner 1 and 2, 1 and 3 and so forth are contiguous in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}\}$  where  $e_1 = (1,2)$   $e_2 = (1,3)$   $e_3 = (1,5)$   $e_4 = (1,6)$   $e_5 = (2,3)$   $e_6 = (2,5)$   $e_7 = (2,7)$   $e_8 = (3,6)$   $e_9 = (3,7)$   $e_{10} = (5,6)$   $e_{11} = (5,7)$   $e_{12} = (6,7)$ .



Subsequently, the  $N[G]$  of  $G(R)$  nbd diagram is a 4-Regular chart.

**End:** If  $n$  is even or odd, the nbd chart  $N[G]$  of  $G(R)$  is a Regular diagram, as displayed in the graph above.

**The relationship between  $n$  and  $N[G]$  is seen in the table below.**

| Number of vertices<br>$n \geq 5$ | $N[G] = (n-3)/2$ - Regular<br>graph | $N[G] = (n-4)/2$ - Regular<br>graph |
|----------------------------------|-------------------------------------|-------------------------------------|
|                                  | $n$ is odd ( $n \geq 5$ )           | $n$ is even ( $n \geq 6$ )          |
|                                  | 2-Regular graph                     | 2-Regular graph                     |
| 5                                | 2-Regular graph                     |                                     |
| 6                                |                                     | 2-Regular graph                     |
| 7                                | 4-Regular graph                     |                                     |
| 8                                | .                                   | 4-Regular graph                     |
| .                                | .                                   | .                                   |
| .                                | .                                   | .                                   |
| .                                | .                                   | .                                   |

**Hypothesis :** The ring of numbers modulo  $n$ ,  $R = Z_n$ , fulfills every one of the standards in hypothesis 3.2.1.  $N[G] = [V(N[G], E(N[G]))]$  where  $E(N[G]) = u, v \in G(R) / u$  and  $v$  are contiguous iff  $nbd(u) \cap nbd(v) = N[G]$  is then a Regular chart.

**Confirmation:** Given  $Z_n$ , the ring of whole numbers modulo  $n$ , and  $R = Z_n Z_n$ ,  $((R, +n, n))$  is a ring to such an extent that  $R = Z_n Z_n = (a, b) / a, b \in Z_n$  with expansion modulo  $n$ , ' $+n$ ' and increase

modulo n, 'n'.

Let  $x, y \in R$  and  $x, y \in R$ , then  $x = (x_1, x_2)$   $y = (y_1, y_2)$  where  $x_1, x_2, y_1, y_2 \in R$ . Now  $x$  is adjacent to  $y$

$\square x + y = (0,0)$  and  $x, y \in (0,0)$

$\square (x_1, x_2) +_n (y_1, y_2) = (0,0)$

$\square (x_1 + y_1, x_2 + y_2) = (0,0)$

$\square x_1 + y_1 = 0, x_2 + y_2 = 0 \quad \square$  If  $x$  has an additive inverse  $y$  then  $x$  and  $y$  are adjacent.

The diagram of  $R$ , for example  $G(R)$ , is a bipartite diagram if  $R$  meets every one of the conditions in hypothesis 3.2.1.

When  $n=2r+1$  ( $r \geq 1$ ), for example ( $n=3$ ),  $G(R)$  is a  $(2r(r+1), 2r(r+1))$ - bipartite chart in  $ZnZn$ .

In case  $n$  is positive,

Then, at that point,  $R-(0,0)$  8 (i.e.,  $G(R)$  has somewhere around 8 vertices subsequent to taking out the  $(0,0)$  component), which is apportioned into two segments, each with  $2r(r+1)$  4 vertices.

Presently we need to discover  $G$ 's nbd chart  $N(G)(R)$ .

By definition,  $x$  and  $y$  are close by in  $N(G)$  for each two qualities  $x, y \in G(R)$  iff  $nbd(u) \cap nbd(v) = \emptyset$ .

Since  $G(R)$  is a bipartite chart with two segments,  $X$  and  $Y$ , no two vertices of  $X$  (or  $Y$ ) are adjoining, and each vertex of  $X$  is close to one vertex of  $Y$ .

Subsequently, every pair of vertices  $x$  and  $y$  in the nbd chart  $N(G)$  is adjoining in  $G(R)$ . The nbd of a vertex in  $X$  (or  $Y$ ) is a singleton set since each vertex in  $X$  (or  $Y$ ) is contiguous only one vertex in  $Y$  (or  $X$ ). Accordingly, the crossing point of any two vertices in  $X$  and  $Y$  with nbd is unfilled. Subsequently, any two  $G(R)$  vertices  $x$  and  $y$  that are not nearby in  $G(R)$  are neighboring in the chart  $N(G)$ . For instance, if  $n=3$ ,  $R=9$ .  $G(R)$  is a  $(4,4)$  bipartite chart with the property that

$V(G(R))=8$  (excluding the  $(0,0)$  vertex).

Then, at that point,  $V(N(G))= 8$  and each  $u,v$  in  $N(G)$  is adjoining.  $N(G)$  is a 6-standard chart.

Proceeding thusly, if  $n=2r+1$  and  $(r \ 0)$ ,  $N(G)$  is a  $(n^2 - 3)$ - customary diagram.

Leave  $R=Z_n$  alone a commutative ring of numbers modulo  $n$ , where  $n$  is more noteworthy than three.

Case (I): Let  $n=9$  then  $R= \{(0,0) (0,1) (0,2) (1,0) (1,1) (1,2) (2,0) (2,1) (2,2)\}$  and  $G(R)$  is a

(4,4)-Bipartite graph with  $V(G(R))=\{(0,1), (0,2), (1,0), (2,0), (1,1), (2,2), (1,2), (2,1)\}$ . Now

$$\text{nbd } [(0,1)] = \{(0,1), (0,2)\} \quad \text{nbd } [(1,0)] = \{(1,0), (2,0)\}, \quad \text{nbd } [(1,1)] = \{(1,1), (2,2)\}, \quad \text{nbd } [(1,2)] = \{(1,2), (2,1)\}, \quad \text{nbd } [(2,0)] = \{(0,1), (0,2)\}, \quad \text{nbd } [(2,1)] = \{(1,0), (2,0)\}, \quad \text{nbd } [(2,2)] = \{(1,1), (2,2)\}.$$

$$\text{nbd } [(0,1)] \cap \text{nbd } [(1,0)] = \emptyset, \quad \text{nbd } [(0,1)] \cap \text{nbd } [(1,1)] = \emptyset, \quad \text{nbd } [(0,1)] \cap \text{nbd } [(2,0)] = \emptyset, \quad \text{nbd } [(0,1)] \cap \text{nbd } [(2,1)] = \emptyset, \quad \text{nbd } [(0,1)] \cap \text{nbd } [(2,2)] = \emptyset.$$

$$\text{nbd } [(1,1)] = \emptyset \text{ and so on. Therefore } (0,1) \text{ and } (2,0), (1,0) \text{ and } (2,2) \text{ etc. are adjacent in } N[G].$$

.Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10},\}$

$e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24}, e_{25}\}$  where  $e_1 = [(0,1), (1,0)]$   
 $e_2 = [(0,1), (2,0)]$

$e_3 = [(0,1), (1,1)]$

$e_4 = [(0,1), (2,2)]$

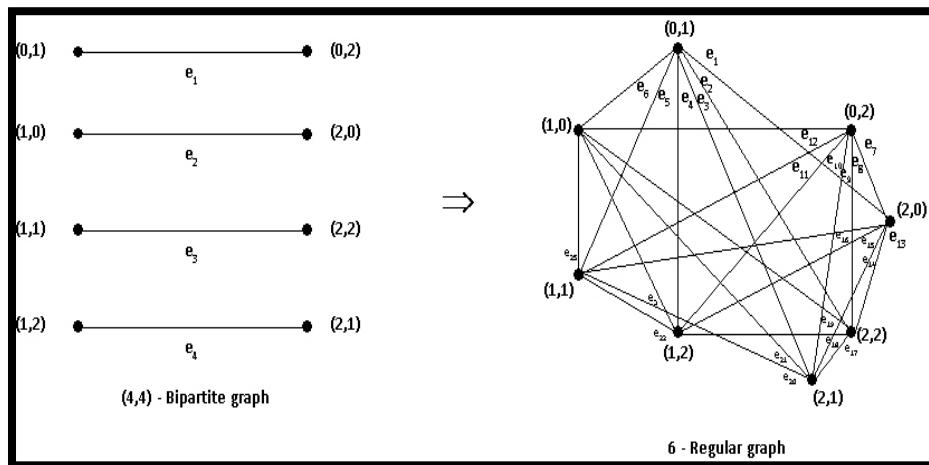
$e_5 = [(0,1), (1,2)]$

$e_6 = [(0,1), (2,1)] e_7 = [(0,2), (1,0)] e_8 = [(0,2), (2,0)] e_9 = [(0,2), (1,1)] e_{10} = [(0,2), (2,2)] e_{11} = [(0,2), (1,2)] e_{12} = [$

$(0,2), (2,1)] e_{13} = [(1,0), (2,0)] e_{14} = [(1,0), (1,1)] e_{15} = [(1,0), (2,2)] e_{16} = [(1,0), (2,1)] e_{17} = [(1,0), (1,2)] e_{18} = [$

$(2,0), (1,1)] e_{19} = [(2,0), (2,2)] e_{20} = [(2,0), (1,2)] e_{21} = [(2,0), (2,1)] e_{22} = [(1,1), (1,2)]$

$e_{23} = [(1,1), (2,1)] e_{24} = [(2,2), (1,2)] e_{25} = [(2,2), (2,1)].$



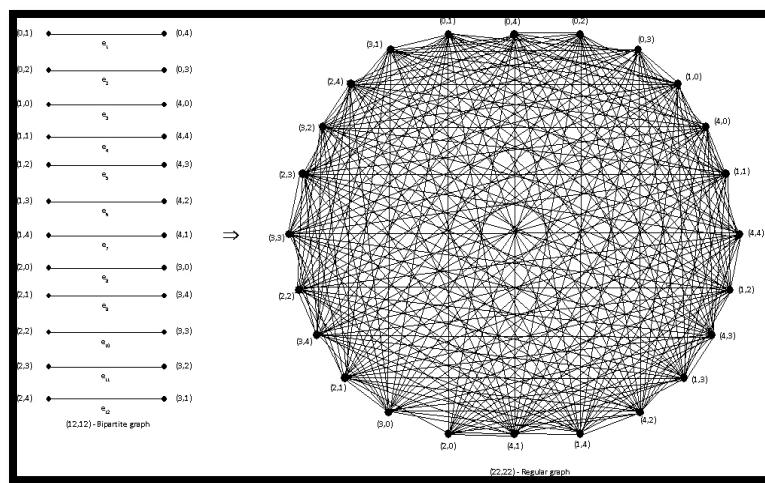
Thusly the nbd chart  $N[G]$  of  $G(R)$  is a 6-Regular diagram. Case(ii):Let  $n=5$  then  $R=\{(0,0)(0,1)(0,2)(0,3)(0,4)(1,0)(1,2)(1,3)(1,4)(2,0)(2,1)(2,2)(2,3)(2,4)(3,0)$

$(3,1)(3,2)(3,3)(3,4)(4,0)(4,1)(4,2)(4,3)(4,4)\}$  and  $G(R)$  is a  $(12,12)$ - Bipartite diagram with  $V(G(R))=\{(0,1),(0,2),(1,1),(1,2),(1,3),(1,4),(2,0),(2,1),(2,3),(2,0,4),(0,3),(4,0),(4,4),(4,3),(4,1),(3,0),(3,4),(3,3),(3,2),(3,1)\}$ . Now,  $nbd[(0,1)] = \{(0,1), (0,4)\}$  nbd

$$\begin{aligned}
 [(0,2)] &= \{(0,2), (-0,3)\} \quad \text{nbd } [(1,0)] = \{(1,0), (-4,0)\} & \text{nbd } [(1,1)] &= \{(1,1), (-4,4)\} \\
 & & \text{nbd } [(1,2)] &= \{(1,2), (-4,3)\} \\
 & & \text{nbd } [(1,3)] &= \{(1,3), (-4,2)\} \quad \text{nbd } [(1,4)] = \{(1,4), (-4,1)\} & \text{nbd } [(2,0)] &= \{(2,0), (-3,0)\} \\
 & & \text{nbd } [(2,1)] &= \{(2,1), (-3,4)\} \quad \text{nbd } [(2,2)] = \{(2,2), (-3,3)\} \quad \text{nbd } [(2,3)] = \{(2,3), (-3,2)\} & \text{nbd } \\
 & & [(2,4)] &= \{(2,4), (-3,1)\} \quad \text{nbd } [(0,4)] = \{(0,1), (-0,4)\} \quad \text{nbd } [(0,3)] = \{0,2), (-0,3)\} \quad \text{nbd } [(4,0)] \\
 & & = \{(1,0), (-4,0)\} \quad \text{nbd } [(4,4)] = \{(1,1), (-4,4)\} \quad \text{nbd } [(4,3)] = \{(1,2), (-4,3)\} \quad \text{nbd } [(4,2)] = \{(1,3), (-4,2)\} \\
 & & \text{nbd } [(4,1)] &= \{(1,4), (-4,1)\} \quad \text{nbd } [(2,0)] = \{(2,0), (-3,0)\} \quad \text{nbd } [(3,4)] = \{(2,1), (-3,4)\} & \text{nbd } \\
 & & [(3,3)] &= \{(2,2), (-3,3)\} \quad \text{nbd } [(3,2)] = \{(2,3), (-3,2)\} & \text{nbd } [(3,1)] &= \{(2,4), (-3,1)\}.
 \end{aligned}$$

$[(0,1)] \cap \text{nbd } [(0,3)] = \emptyset$ ,  $\text{nbd } [(0,1)] \cap \text{nbd } [(4,0)] = \emptyset$ ,  $\text{nbd } [(0,1)] \cap \text{nbd } [(4,4)] = \emptyset$  thus on. Therefore  $(0,1)$  and  $(0,2)$ ,  $(0,2)$  and  $(4,0)$  and so forth are adjoining in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}\}$  where  $e_1 = [(0,1), (0,3)]$   $e_2 = [(0,1), (4,0)]$

e3=[(0,1),(4,4)] ect.



Therefore, the  $N[G]$  of  $G(R)$  nbd diagram is a 22-Regular chart.

**End:** If  $n$  is odd, the nbd chart  $N[G]$  of  $G(R)$  is a  $(n^2 - 3)$  - Regular diagram, as found in the

previous picture.

The relationship between  $n$  and  $N[G]$  is seen in the table below.

| <b>r</b> | <b>Number of vertices</b><br>$n=2r+1(n \neq 3)$ | $ R  = n^2$ | $N[G] = (n^2-3)$ -Regular graph |
|----------|---|-------------|---------------------------------|
| 1        | 3   | 9           | 6- Regular graph                |
| 2        | 5   | 25          | 22- Regular graph               |
| 3        | 7   | 49          | 47-Regular graph                |
| .        | .   | .           | .                               |
| .        | .   | .           | .                               |
| .        | .   | .           | .                               |
| .        | .   | .           | .                               |

### Triangle neighbourhood graphs and complete bipartite graphs on $Z_n$ and $Z_n$

#### $Z_n$

This segment talks about the local diagrams (curtailed nbd charts) of the triangle and full bipartite diagrams  $Z_n$  and  $Z_n$ . It is sure that the got nbd charts are finished diagrams.

Hypothesis 5.3.1: The ring of whole numbers modulo  $n$ ,  $R=Z_n$ , fulfills each of the measures in hypothesis 2.2.7.

Characterize  $N[G]=[V(N[G],E(N[G]))]$  as the local chart of  $G(R)$ , where  $V(N[G])$  is the vertex set and  $E(N[G])$  is the edge set, and  $E(N[G])=u,v \in G(R) \wedge u \text{ and } v \text{ are nearby iff } \text{nbd}(u) \cap \text{nbd}(v) \neq \emptyset$ .  $N[G]$  is then a Complete diagram.

Leave  $R=Z_n$  alone the ring of whole numbers modulo  $n$  that fulfills every one of the states of hypothesis 2.3.1. The diagram  $G(R)$  is then a triangle chart where  $n \neq 3$  or 4. In case  $n$  is an odd number ( $n \neq 3$ ), the diagram  $G(R)$  is  $((n-1)/2)$  – triangle chart. Essentially, in case  $n$  is a much number ( $n \neq 4$ ), the chart  $G(R)$  is a  $((n-2)/2)$  – three-sided diagram. Since  $G(R)$  is a three-sided diagram, every triangle has definitively one vertex  $x,y,z$  that is zero, and each vertex of  $G(R)$

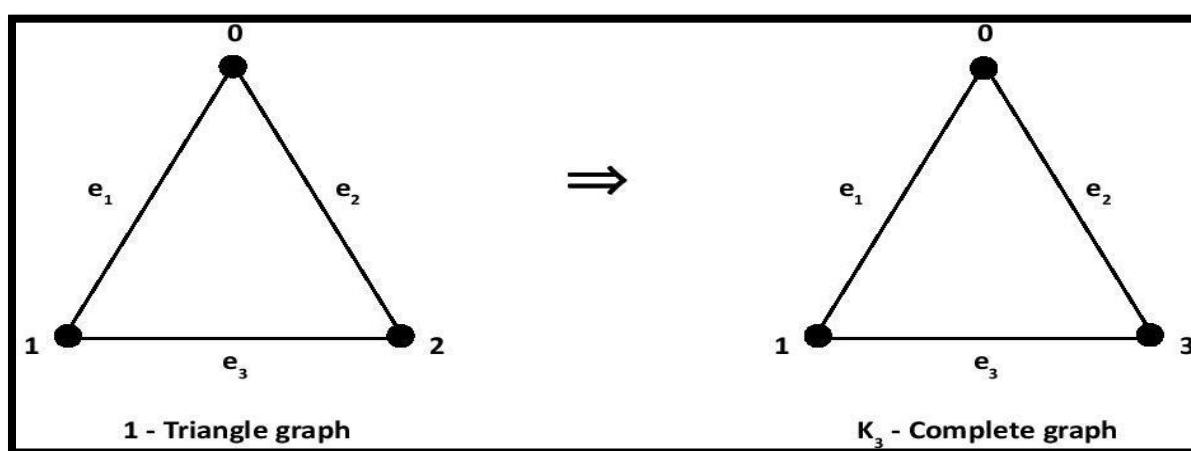
has a level of somewhere around 2. Subsequently, in the nbd diagram  $N(G)$  of  $G(R)$ , for each two vertices  $u$  and  $v$  in  $G(R)$ , if  $nbd(u) \cap nbd(v) \neq \emptyset$  then  $nbd(u) \cap nbd(v) \neq \emptyset$  and  $nbd(v) \cap nbd(u) \neq \emptyset$

In the event that  $u$  and  $v$  are nearby in  $G(R)$ , they are moreover neighboring in  $N(G)$ . Because vertex zero is the normal vertex for all triangles, vertices that are not adjoining in  $G(R)$  are similarly nearby in  $N(G)$ .

Therefore,  $nbd(u)$  and  $nbd(v)$  incorporate the vertex zero for any two vertices  $u$  and  $v$ . As an outcome,  $nbd(u) \cap nbd(v) \neq \emptyset$  in  $N(G)$ , each two vertices  $u$  and  $v$  of  $G(R)$  are contiguous. Thus,  $N(G)$  is a Complete diagram. Proceeding in this vein, assuming  $n$  is odd ( $n=3$ ) or even ( $n=4$ ) the nbd diagram  $N[G]$  is a  $K_n$ -Complete chart or a  $K_{n-1}$ -Complete diagram. 5.3.2 Illustration: Consider  $R=\mathbb{Z}_n$  to be a commutative ring of numbers modulo  $n$ , where  $n=3$ .

Case(i): Let  $n=3$  then  $R=\{0,1,2\}$  and the diagram  $G(R)$  is a 1-triangle chart with  $V(G(R))=\{0,1,2\}$ . Now  $nbd[0]=\{0,1,2\}$   $nbd[1]=\{0,1,2\}$   $nbd[2]=\{0,1,2\}$ . Since  $nbd$

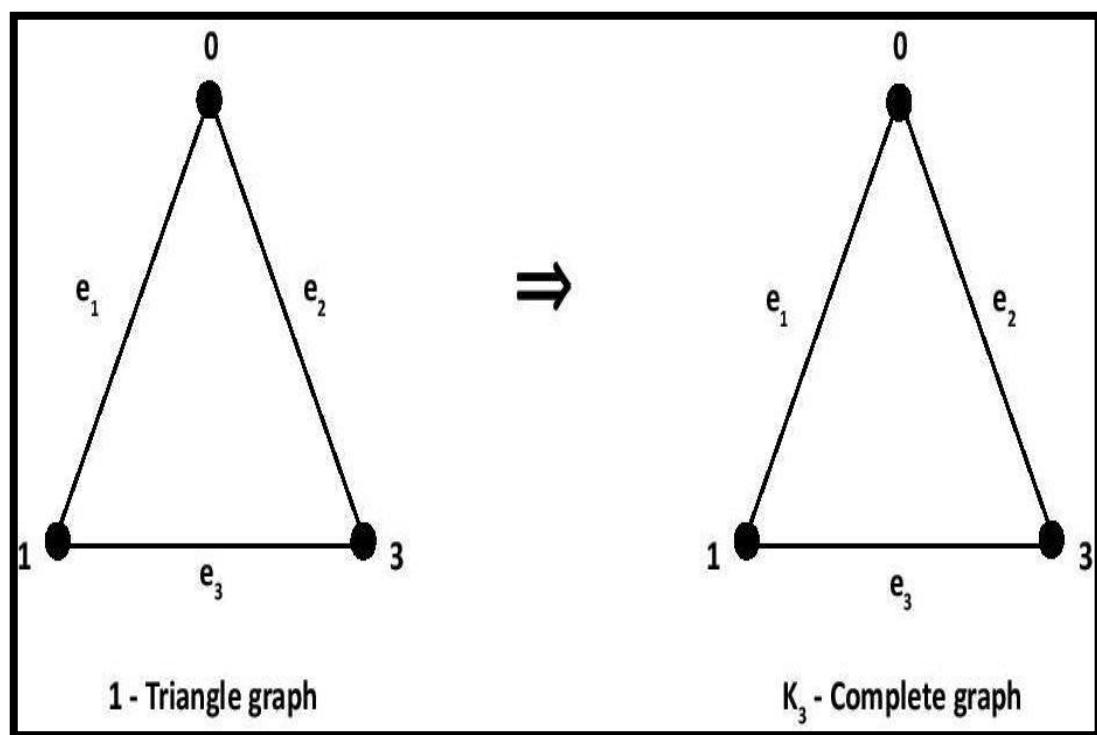
$[0] \cap nbd[1] \neq \emptyset$ ,  $nbd[0] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[2] \neq \emptyset$ . Hence 0 and 1, 0 and 2 and so forth are contiguous in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3\}$  where  $e_1=(0,1)$ ,  $e_2=(0,2)$ ,  $e_3=(1,2)$



Accordingly the nbd diagram  $N[G]$  of  $G(R)$  is a  $K_3$ -Complete chart.

Additionally Let  $n=4$  then  $R= \{0,1,2,3\}$  and the diagram  $G(R)$  is a 1-triangle chart with  $V(G(R))=\{0,1,3\}$ .Now  $nbd[0]=\{0,1,3\}$        $nbd[1]=\{0,1,3\}$        $nbd[3]=\{0,1,3\}$ .Since  $nbd[0]$

$\cap$   $nbd[1] \square \phi$ ,  $nbd[0] \cap nbd[3] \square \phi$ ,  $nbd[1] \cap nbd[3] \square \phi$ . Along these lines 0 and 1,0 and 3 and so on are adjoining in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3\}$  where  $e_1 = (0,1)$ ,  $e_2 = (0,3)$ ,  $e_3 = (1,3)$ .

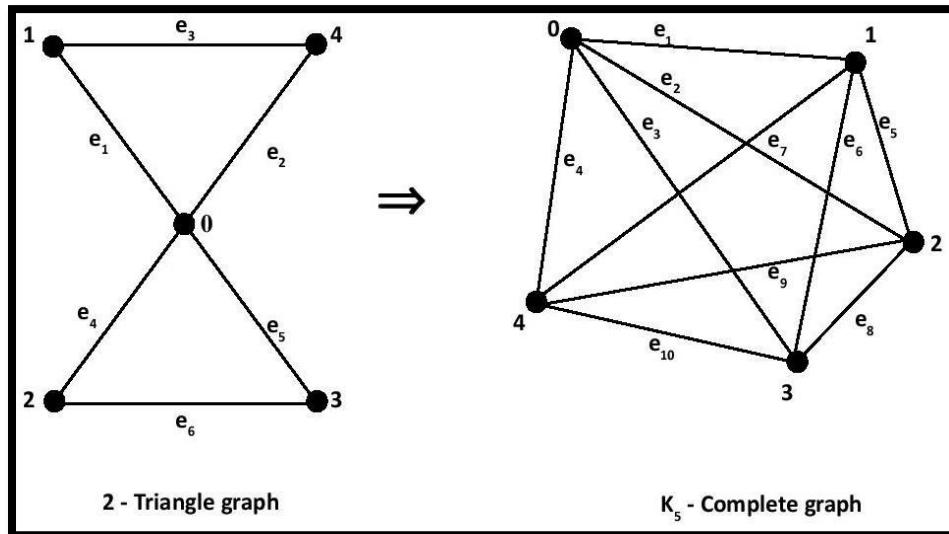


Hence the nbd diagram  $N[G]$  of  $G(R)$  is a K3-Complete chart.

Case(ii): Let  $n=5$  then  $R = \{0,1,2,3,4\}$  and the graph  $G(R)$  is a 2-triangle diagram with  $V(G(R)) = \{0,1,2,3,4\}$ . Now  $nbd[0] = \{0,1,2,3,4\}$        $nbd[1] = \{0,4\}$   $nbd[2] = \{0,3\}$   $nbd[3] = \{0,2\}$   $nbd[4] = \{0,1\}$ . Since  $nbd[0] \cap nbd[1] \neq \emptyset$ ,  $nbd[0] \cap nbd[2] \neq \emptyset$ ,  $nbd[0] \cap nbd[3] \neq \emptyset$ ,  $nbd[0] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ . In this manner 0 and 1, 0 and 2 and so on are adjoining in  $N[G]$ . Hence  $E(N[G]) = \{e1, e2, e3, e4, e5, e6\}$  where  $e1 = (0,1)$   $e2 = (0,2)$   $e3 = (0,3)$   $e4 = (0,4)$   $e5 = (1,2)$   $e6 = (1,3)$   $e7 = (1,4)$   $e8 = (2,3)$   $e9 = (2,4)$   $e10 = (3,4)$ .

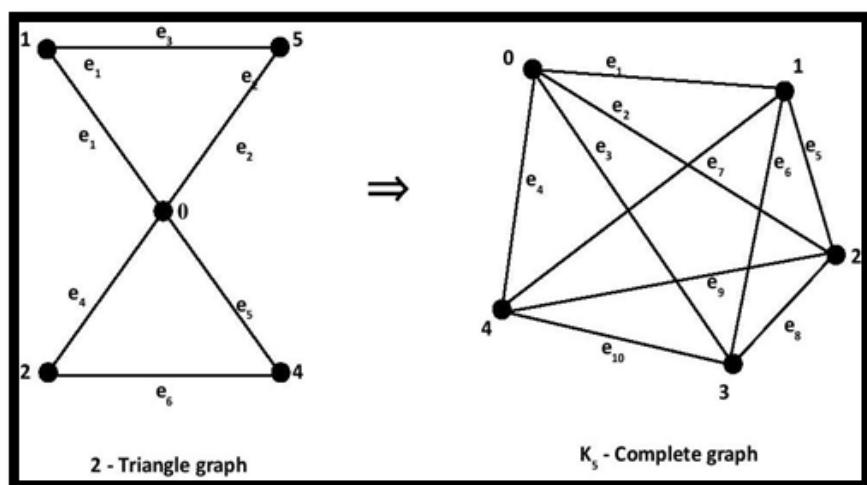
Thusly the nbd diagram  $N[G]$  of  $G(R)$  is a  $K_5$ -Complete chart.

Also Let  $n=6$ , then  $R = \{0,1,2,3,4,5\}$  and the diagram  $G(R)$  is a 2-triangle chart with  $V(G(R)) = \{0,1,2,3,4,5\}$ . Now  $nbd[0] = \{0,1,2,4,5\}$     $nbd[1] = \{0,1,5\}$     $nbd[2] = \{0,2,4\}$     $nbd[3] = \{1,2,3\}$     $nbd[4] = \{0,4,5\}$     $nbd[5] = \{1,4,5\}$ .



$nbd[4] = \{0,2,4\}$     $nbd[5] = \{0,1,5\}$ . Since  $nbd[0] \cap nbd[1] \neq \emptyset$ ,  $nbd[0] \cap nbd[2] \neq \emptyset$ ,  $nbd[0] \cap nbd[3] \neq \emptyset$ ,  $nbd[0] \cap nbd[4] \neq \emptyset$ ,  $nbd[0] \cap nbd[5] \neq \emptyset$ ,  $nbd[1] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[5] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[5] \neq \emptyset$ . Therefore 0 and 2, 0 and 5 and so forth are contiguous in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3\}$     $e_1 = (0,1)$     $e_2 = (0,2)$     $e_3 = (0,4)$     $e_4 = (0,5)$     $e_5 = (1,2)$     $e_6 = (1,5)$     $e_7 = (2,4)$     $e_8 = (2,5)$     $e_9 = (3,4)$     $e_{10} = (4,5)$ .

Therefore, the  $N[G]$  nbd chart of  $G(R)$  is a  $K_5$ -Complete diagram



**End:** We can see from the past outline that in case  $n$  is even or odd, the nbd chart  $N[G]$  of  $G(R)$  is a Complete diagram.

The relationship between  $n$  and  $N[G]$  is seen in the table below.

| Number of vertices<br>$n \geq 3$ | $G(R) = K_n$ -Complete graph<br>$n$ is odd ( $n \geq 3$ ) | $G(R) = K_{n-1}$ -Complete graph<br>$n$ is even ( $n \geq 4$ ) |
|----------------------------------|---|--|
|                                  |   |  |
| 3                                | $K_3$ -Complete graph                                     |  |
| 4                                |   | $K_3$ -Complete graph  |
| 5                                | $K_5$ -Complete graph                                     |  |
| 6                                |   | $K_5$ -Complete graph  |
| .                                | .   | .  |
| .                                | .   | .  |

Leave  $R = Z_n$  alone the ring of whole numbers modulo  $n$ , which meets every one of the standards in hypothesis 2.3.3. Characterize  $N[G] = [V(N[G], E(N[G]))]$  as the local chart of  $G(R)$ , where  $V(N[G])$  is the vertex set and  $E(N[G])$  is the edge set, and  $E(N[G]) = u, v G(R) / u$  and  $v$  are adjoining iff  $nbd(u) \cap nbd(v)$ .  $N[G]$  is then a Complete diagram.

Leave  $R = Z_n$  alone the ring of whole numbers modulo  $n$  that fulfills every one of the states of hypothesis 2.3.3. The diagram  $G(R)$  is then a triangle chart with  $n = 3$ . In case  $n$  is a different of 2(or)3 (which are not prime). The chart  $G(R)$  is then a triangle diagram.  $G(R)$  is a triangle diagram, and every vertex on  $G(R)$  has a level of no less than 2. Subsequently, in the nbd chart  $N(G)$  of  $G(R)$ , for each two vertices  $u$  and  $v$  in  $G(R)$ , if  $nbd(u) \cap nbd(v) \neq \emptyset$  then  $nbd(u) \cap nbd(v) \neq \emptyset$

For each two adjoining vertices  $u$  and  $v$  in  $G(R)$ , they are additionally neighboring in  $N(G)$ . Also, the vertices that are not bordering in  $G(R)$  are neighboring in  $N(G)$ . As an outcome,  $nbd(u) \cap nbd(v)$  incorporate the normal vertex 0 for any two vertices  $u$  and  $v$ . Hence  $nbd(u) \cap nbd(v) \neq \emptyset$ . That is, in  $N[R]$ , each two vertices  $u$  and  $v$  of  $G(R)$  are adjoining. Thus,  $N(G)$  is a

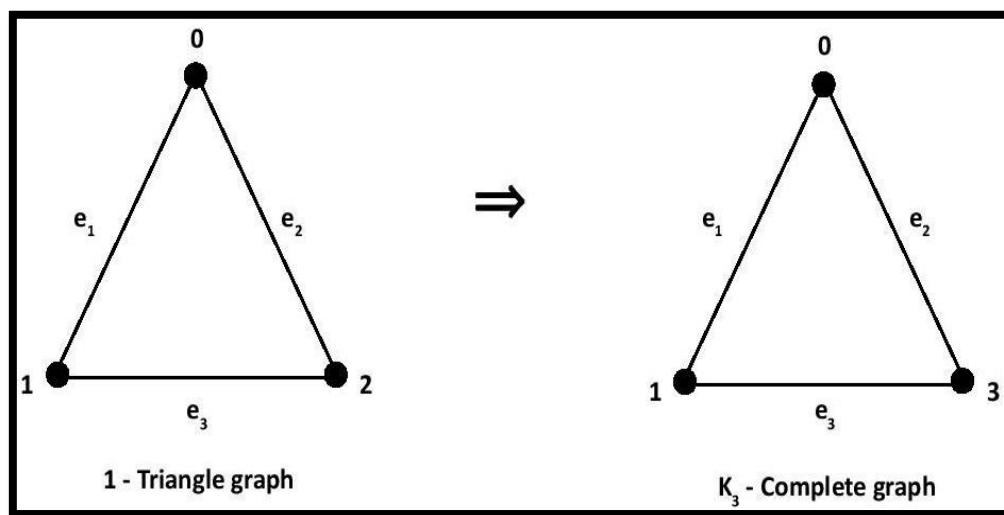
Complete graph. Continuing in same vein, in case  $n$  is a number of 2(or)3, the nbd chart  $N[G]$  is a Complete diagram.

Accept  $R = \mathbb{Z}_n$  is a commutative ring of whole numbers modulo  $n$ , where  $n = 3$ .

Case I If  $n$  is 3, then, at that point,  $R$  is 0, 1, 2, and the chart  $G(R)$  is a 1-triangle diagram with  $V(G(R)) = 0, 1, 2$ .

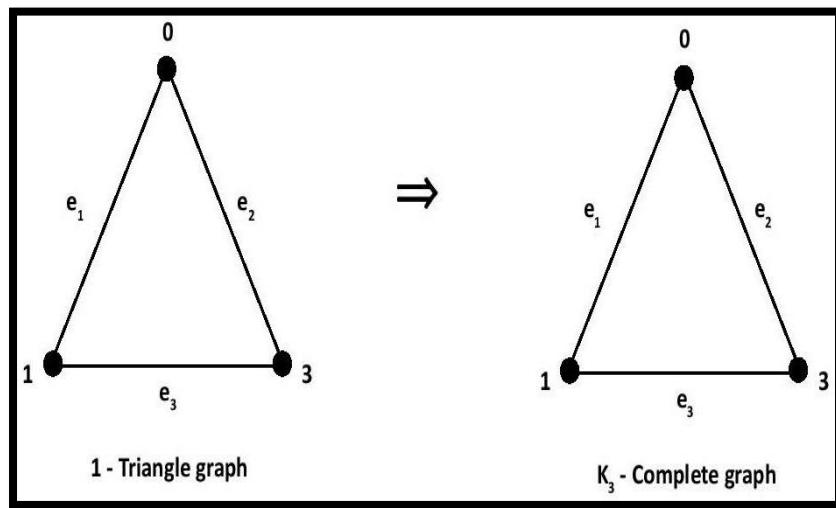
Presently, nbd [0] = 0, 1, 2 nbd [1] = 0/1,2 nbd [2] = 0/1,2 Since nbd [0] nbd [1], nbd [0] nbd [2], nbd [1] nbd [2], nbd [1] nbd [2]. Accordingly, in  $N[G]$ , 0 and 1, 0 and 2, etc are nearby. Accordingly,  $E(N[G]) = e_1, e_2, e_3$ , where  $e_1 = (0,1)$ ,  $e_2 = (0,2)$ , and  $e_3 = (1,2)$ .

Consequently the nbd diagram  $N[G]$  of  $G(R)$  is a  $K_3$ -Complete graph. Similarly Let  $n=4$ , then  $R = \{0,1,2,3\}$  and the chart  $G(R)$  is a 1-triangle chart,  $V(G(R)) = \{0,1,3\}$ . nbd[0] {0,1,3} nbd [1] = {0,1,3} nbd [3] = {0,1,3}. Since nbd [0]  $\cap$  nbd [1]  $\neq \emptyset$ , nbd [0]  $\cap$  nbd [3]  $\neq \emptyset$ , nbd [1]  $\cap$  nbd [3]  $\neq \emptyset$ . Consequently 0 and 1, 0 and 3 and so forth are nearby in  $N[G]$ . Hence  $G = \{e_1, e_2, e_3\}$  where  $e_1 = (0,1)$   $e_2 = (0,3)$   $e_3 = (1,3)$ .

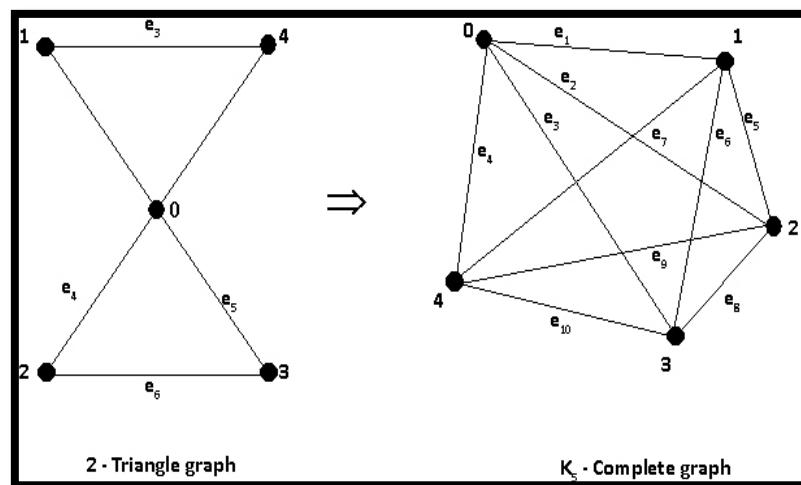


Accordingly the nbd chart  $N[G]$  of  $G(R)$  is a  $K_3$ -Complete diagram.

Case(ii): Let  $n=5$  then  $R= \{0,1,2,3,4\}$  and the chart  $G(R)$  is a 2-triangle diagram with



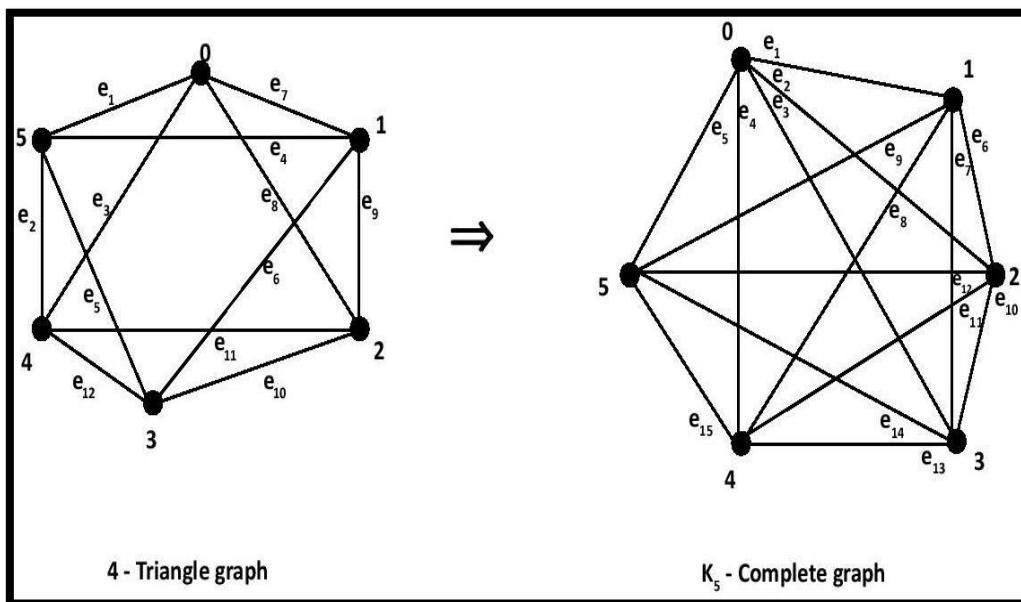
$V(G(R))=\{0,1,2,3,4\}$ . Now  $nbd[0]=\{0,1,2,3,4\}$   $nbd[1]=\{0,4\}$   $nbd[2]=\{0,3\}$   $nbd[3]=\{0,2\}$ . Since  $nbd[4]=\{0,1\}$ .  $nbd[0] \cap nbd[1] \neq \emptyset$ ,  $nbd[0] \cap nbd[2] \neq \emptyset$ ,  $nbd[0] \cap nbd[3] \neq \emptyset$ ,  $nbd[0] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ . Accordingly 0 and 1, 0 and 2 and so on are contiguous in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6\}$  where  $e_1=(0,1)$   $e_2=(0,2)$   $e_3=(0,3)$   $e_4=(0,4)$   $e_5=(1,2)$   $e_6=(1,3)$   $e_7=(1,4)$   $e_8=(2,3)$   $e_9=(2,4)$   $e_{10}=(3,4)$ .



Thusly the nbd chart  $N[G]$  of  $G(R)$  is a  $K_5$ -Complete diagram.

Also Let  $n=6$  then  $R= \{0,1,2,3,4,5\}$  and the chart  $G(R)$  is a 2-triangle diagram with  $V(G(R))=\{1,2,3,4,5\}$ . Presently  $nbd[0]=\{0,1,2,4,5\}$

$=\{0,1,2,3,4\}$   $nbd[3]=\{0,1,2,3,4,5\}$   $nbd[4]=\{0,2,3,4,5\}$   $nbd[5]=\{0,1,3,4,5\}$ . Since  $nbd[0] \cap nbd[1] \neq \emptyset$ ,  $nbd[0] \cap nbd[2] \neq \emptyset$ ,  $nbd[0] \cap nbd[3] \neq \emptyset$ ,  $nbd[0] \cap nbd[4] \neq \emptyset$ ,  $nbd[0] \cap nbd[5] \neq \emptyset$ ,  $nbd[1] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[5] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[5] \neq \emptyset$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[5] \neq \emptyset$ ,  $nbd[4] \cap nbd[5] \neq \emptyset$ . Along these lines 0 and 1, 0 and 2 and so forth are contiguous in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{13}, e_{14}, e_{15}\}$  where  $e_1=(0,1)$   $e_2=(0,2)$   $e_3=(0,3)$   $e_4=(0,4)$   $e_5=(0,5)$   $e_6=(1,2)$   $e_7=(1,3)$   $e_8=(1,4)$   $e_9=(1,5)$   $e_{10}=(2,3)$   $e_{11}=(2,4)$   $e_{12}=(2,5)$   $e_{13}=(3,4)$   $e_{14}=(3,5)$   $e_{15}=(4,5)$ .



$G(R)$ 's chart  $N[G]$  is a  $K_6$ -Complete diagram.

**End:** Based on the above outline, we can derive that assuming  $n$  is a various of 2 or 3,

$G(R)$  nbd chart  $N[G]$  is a Complete diagram.

Leave  $R=\mathbb{Z}_n$  alone the ring of whole numbers modulo  $n$ , which meets every one of the measures in hypothesis 2.3.5.

Characterize  $N[G]=[V(N[G], E(N[G]))]$  as the local chart of  $G(R)$ , where  $V(N[G])$  is the vertex set and  $E(N[G])$  is the edge set, and  $E(N[G])=u, v \in G(R) / u$  and  $v$  are nearby iff  $nbd(u) \cap nbd(v)$ .

$N[G]$  is then a Complete chart.

Evidence: Let  $R = \mathbb{Z}_n$  indicate the ring of whole numbers modulo  $n$  that satisfies every one of the states of hypothesis 2.3.5. The diagram  $G(R)$  is then a triangle chart when  $n = 6$ .

The nbd of a component in the chart  $G(R)$  is characterized as  $nbd(v) =$  The arrangement of all vertices adjoining  $v$  (counting  $v$ ).

Since  $G(R)$  is a triangle diagram  $G(R)$  (where  $n = 6$ ), we might get the nbd chart  $N[G]$  for this triangle chart  $G(R)$ .

By where the edge set of  $N[G]$  is characterized as  $E(N[G]) = u, v \in G(R) / nbd(u) \text{ and } nbd(v)$  are adjoining iff  $nbd(u) \cap nbd(v)$

Since  $G(R)$  is a triangle diagram, every vertex has a level of no less than 2, and  $n = 6$ .

Then, at that point, as indicated by the meaning of  $N[G]$ , for each two vertices  $u, v \in G(R)$ , at least one vertex is normal to the  $nbd(u)$  and  $nbd(v)$ , i.e.  $nbd(u) \cap nbd(v)$ . Subsequently,  $u$  and  $v$  are close by in  $N[G]$ . So in  $N[G]$ , there should be an edge between each pair of vertices, or each pair of vertices in  $N[G]$  is contiguous. Accordingly,  $N[G]$  is a full chart with  $n$  vertices ( $n = 6$ ).

Proceeding in this vein, if  $n = 2r$  or  $3r$ , where  $r \geq 3$  is a finished diagram, we see that  $N[G]$  is a finished chart.

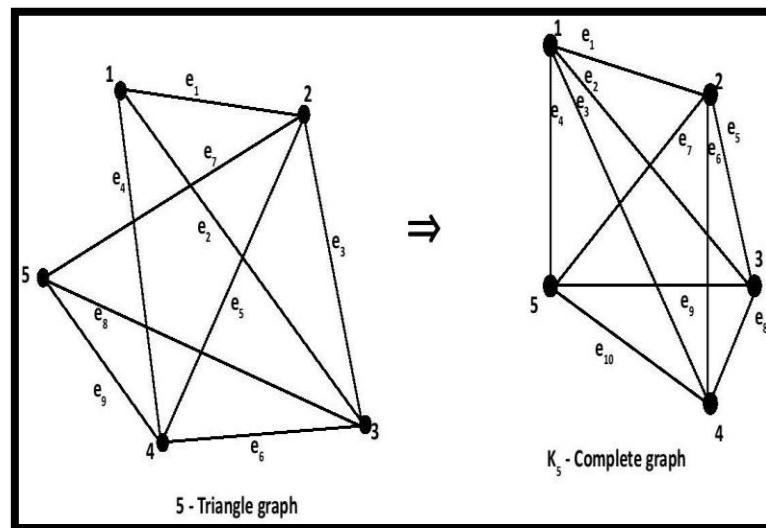
5.3.6 Illustration:  $R = \mathbb{Z}_n$  means a commutative ring of whole numbers modulo  $n$ , where  $n = 6$ .

Case(i): Assume  $n = 6$  and  $R = \{0, 1, 2, 3, 4, 5\}$ , and the diagram is a 5-triangle chart.

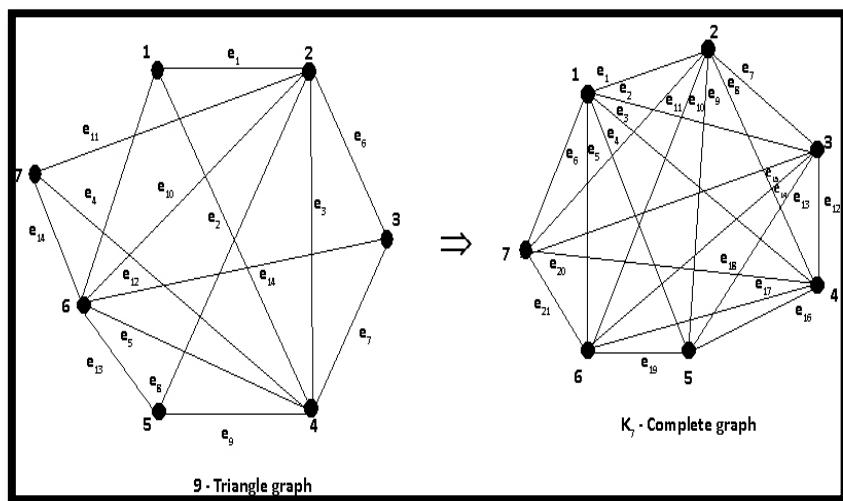
$V(G(R)) = \{1, 2, 3, 4, 5\}$ . Presently  $nbd[1] = \{1, 2, 3, 4\}$   $nbd[2] = \{2, 3, 4, 5\}$   $nbd[3] = \{3, 4, 5\}$   $nbd[4] = \{1, 2, 3, 5\}$   $nbd[5] = \{1, 2, 4, 5\}$ .  
 $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[5] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[5] \neq \emptyset$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[5] \neq \emptyset$ ,  $nbd[4] \cap nbd[5] \neq \emptyset$ . Thusly 1 and 2, 1 and 3 and so on are neighboring in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$  where  $e_1 = (1, 2)$   $e_2 = (1, 3)$   $e_3 = (1, 4)$   $e_4 = (1, 5)$   $e_5 = (2, 3)$   $e_6 = (2, 4)$   $e_7 = (2, 5)$   $e_8 = (3, 4)$   $e_9 = (3, 5)$   $e_{10} = (4, 5)$ .

Hence the nbd diagram  $N[G]$  of  $G(R)$  is a  $K_5$ -Complete chart.

Likewise Let  $n=8$  then  $R= \{0,1,2,3,4,5,6,7\}$  and the chart  $G(R)$  is a 9-triangle diagram with  $V(G(R))=\{1,2,3,4,5,6,7\}$ . Now  $nbd[1]=\{1,2,4,6\}$   $nbd[2]=\{2,3,4,5,6,7\}$   $nbd[3]=\{2,3,4,6\}$   $nbd[4]=\{1,2,3,4,5,6,7\}$   $nbd[5]=\{2,4,5,6\}$   $nbd[6]=\{1,2,4,5,7\}$   $nbd[7]=\{2,4,6,7\}$ . Since  $nbd[1] \cap nbd[2] \neq \emptyset$ ,  $nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[5] \neq \emptyset$ ,  $nbd[1] \cap nbd[6] \neq \emptyset$ ,  $nbd[1] \cap nbd[7] \neq \emptyset$ ,  $nbd[2] \cap nbd[3] \neq \emptyset$ ,  $nbd[2] \cap nbd[4] \neq \emptyset$ ,  $nbd[2] \cap nbd[5] \neq \emptyset$ ,  $nbd[2] \cap nbd[6] \neq \emptyset$ ,  $nbd[2] \cap nbd[7] \neq \emptyset$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[5] \neq \emptyset$ ,  $nbd[3] \cap nbd[6] \neq \emptyset$ ,  $nbd[3] \cap nbd[7] \neq \emptyset$ ,  $nbd[4] \cap nbd[5] \neq \emptyset$ ,  $nbd[4] \cap nbd[6] \neq \emptyset$ ,  $nbd[4] \cap nbd[7] \neq \emptyset$ ,  $nbd[5] \cap nbd[6] \neq \emptyset$ ,  $nbd[5] \cap nbd[7] \neq \emptyset$ ,  $nbd[6] \cap nbd[7] \neq \emptyset$ .



$\square \phi$ ,  $nbd[3] \cap nbd[4] \neq \emptyset$ ,  $nbd[3] \cap nbd[5] \neq \emptyset$ ,  $nbd[3] \cap nbd[6] \neq \emptyset$ ,  $nbd[3] \cap nbd[7] \neq \emptyset$ ,  $nbd[4] \cap nbd[5] \neq \emptyset$ ,  $nbd[4] \cap nbd[6] \neq \emptyset$ ,  $nbd[4] \cap nbd[7] \neq \emptyset$ ,  $nbd[5] \cap nbd[6] \neq \emptyset$ ,  $nbd[5] \cap nbd[7] \neq \emptyset$ ,  $nbd[6] \cap nbd[7] \neq \emptyset$ . In this manner 1 and 1,2 and 3 and so on are neighboring in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}\}$  where  $e_1=(1,2)$   $e_2=(1,3)$   $e_3=(1,4)$   $e_4=(1,5)$   $e_5=(1,6)$   $e_6=(1,7)$   $e_7=(2,3)$   $e_8=(2,4)$   $e_9=(2,5)$   $e_{10}=(2,6)$   $e_{11}=(2,7)$   $e_{12}=(3,4)$   $e_{13}=(3,5)$   $e_{14}=(3,6)$   $e_{15}=(3,7)$   $e_{16}=(4,5)$   $e_{17}=(4,6)$   $e_{18}=(4,7)$   $e_{19}=(5,6)$   $e_{20}=(5,7)$   $e_{21}=(6,7)$ .



Thus,  $G(R_{nbd})$ 's chart  $N[G]$  is a  $K_7$ -Complete bipartite diagram.

**End:** by and large, in case  $n$  is  $2r$  or  $3r$  ( $r \geq 3$ ), we have the nbd chart  $N[G]$  of  $G(R)$ , which is a Complete diagram.

**Hypothesis:** Let  $R = \mathbb{Z}_n$  be the ring of numbers modulo  $n$ , which satisfies every one of the conditions in hypothesis 2.3.7.  $N[G]$  is characterized as  $[V(N[G], E(N[G]))]$ . be the local chart of  $G(R)$ ,  $V(N[G])$  is the vertex set, and  $E(N[G])$  is the edge set, where  $E(N[G]) = u, v \in G(R) / u$  and  $v$  are neighboring iff  $nbd(u) \cap nbd(v) \neq \emptyset$ .  $N[G]$  is then a Complete diagram.

**Leave  $R = \mathbb{Z}_n$**  alone the ring of numbers modulo  $n$  that fulfills each of the measures of hypothesis 2.3.7. The diagram  $G(R)$  is then a triangle chart when  $n = 5$ .

The nbd of a component in the diagram of  $G$  has been characterized ( $R$ ).  $nbd(v) =$  The assortment of all vertices adjoining  $v$  (counting  $v$ ).

At the point when  $n = 5$ , the chart  $G(R)$  is a triangle diagram. We find the nbd diagram  $N[G]$  for this triangle chart  $G(R)$  by characterizing the edge set of  $N[G]$  as  $E(N[G]) = u, v \in G(R) / nbd(u) \cap nbd(v) \neq \emptyset$ .

Since  $G(R)$  is a triangle chart, every vertex has a level of something like 2 and since  $n = 5$ , the meaning of  $N[G]$  states that for any two vertices  $u, v \in G(R)$ , somewhere around one vertex is normal to both.

The nbd ( $u$ ) and nbd( $v$ ), for example  $nbd(u) \cap nbd(v)$  accordingly,  $u$  and  $v$  are close by in  $N[G]$ .

That is, in  $N[G]$ , there should be an edge interfacing each pair of vertices, or each pair of vertices in  $N[G]$  are neighboring. Therefore,  $N[G]$  is a finished chart with  $n$  vertices ( $n \geq 5$ ).

Proceeding in this vein, in case  $n$  is even (odd) ( $n \geq 5$ ), the nbd diagram  $N[G]$  is a Complete chart.

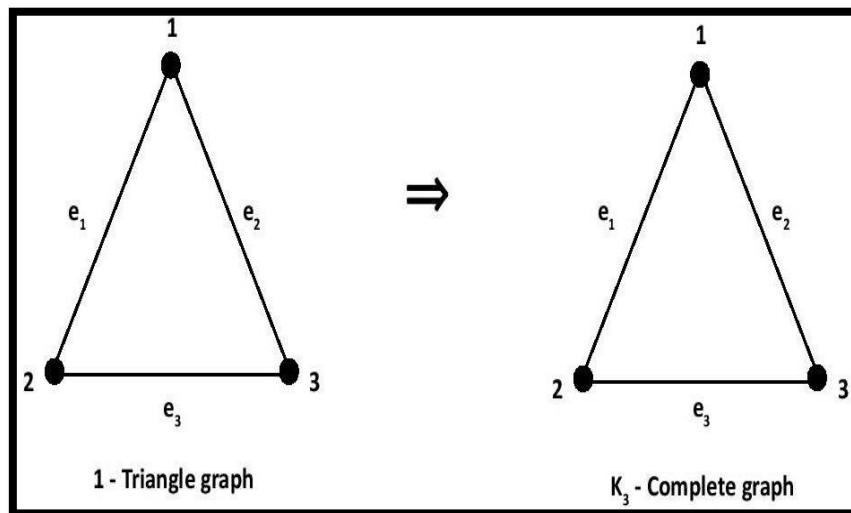
5.3.8 Illustration:  $R=Z_5$  signifies a commutative ring of numbers modulo  $n$ , where  $n = 5$ .

Case I If  $n = 5$ , then, at that point,  $R = 0, 1, 2, 3, 4$ .

$G(R)$  is currently a one-triangle diagram with  $V(G(R)) = \{1, 2, 3\}$ .

$nbd[1] = \{1, 2, 3\}$  now  $nbd[2] = \{1, 2, 3, 4\}$   $nbd[3] = \{1, 2, 3, 5\}$   $nbd[4] = \{1, 2, 3, 6\}$   $nbd[5] = \{1, 2, 3, 7\}$   $nbd[6] = \{1, 2, 3, 8\}$   $nbd[7] = \{1, 2, 3, 9\}$  Since  $nbd[1] \cap nbd[2] = \{1, 2, 3\}$ ,  $nbd[1] \cap nbd[3] = \{1, 2, 3\}$ ,  $nbd[2] \cap nbd[3] = \{1, 2, 3\}$ ,  $nbd[2] \cap nbd[4] = \{1, 2, 3\}$ ,  $nbd[3] \cap nbd[4] = \{1, 2, 3\}$ . Subsequently, in  $N[G]$ , 1 and 2, 1 and 3, etc are contiguous. Subsequently,  $E(N[G]) = e_1, e_2, e_3$ , where  $e_1 = (1, 2)$ ,  $e_2 = (1, 3)$ , and  $e_3 = (2, 3)$ .

Subsequently the nbd chart  $N[G]$  of  $G(R)$  is a  $K_3$ -Complete diagram.



Essentially Let  $n=7$  then  $R= \{0,1,2,3,4,5,6\}$  and the chart is a 4-triangle diagram with  $V(G(R))=\{1,2,3,4,5,6\}$ . Presently  $nbd[1]=\{1,2,4,5\}$   $nbd[2]=\{1,2,4,6\}$   $nbd[3]=\{1,2,3,5\}$

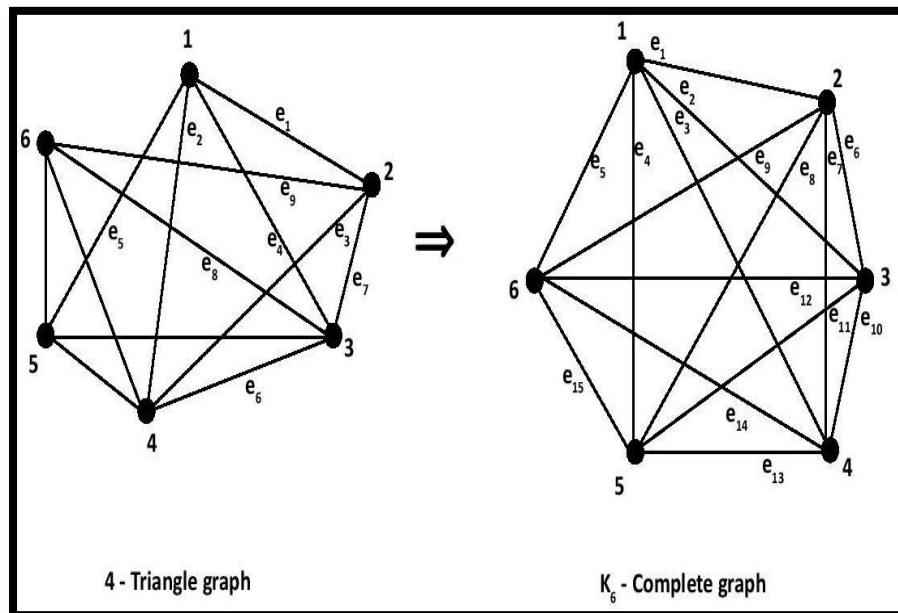
$nbd[4]=\{1,2,3,5,6\}$   $nbd[5]=\{1,3,4,5,6\}$ . Since  $nbd[6]=\{2,3,4,5,6\}$ .  $nbd[1] \cap nbd[2] \neq \emptyset$ ,

$nbd[1] \cap nbd[3] \neq \emptyset$ ,  $nbd[1] \cap nbd[4] \neq \emptyset$ ,  $nbd[1] \cap nbd[5] \neq \emptyset$ ,  $nbd[1] \cap nbd[6] \neq \emptyset$

$\phi, \text{nbd}[2] \cap \text{nbd}[3] \square \phi, \text{nbd}[2] \cap \text{nbd}[4] \square \phi, \text{nbd}[2] \cap \text{nbd}[5] \square \phi, \text{nbd}[2] \cap \text{nbd}[6]$   
 $\square \phi, \text{nbd}[3] \cap \text{nbd}[4] \square \phi, \text{nbd}[3] \cap \text{nbd}[5] \square \phi, \text{nbd}[3] \cap \text{nbd}[6] \square \phi, \text{nbd}[4] \cap \text{nbd}$   
 $[5] \square \phi, \text{nbd}[4] \cap \text{nbd}[6] \square \phi, \text{nbd}[5] \cap \text{nbd}[6] \square \phi$ . Accordingly 1 and 2, 2 and 3 and so forth are

neighboring in  $N[G]$ . Hence  $E(N[G]) = \{e_1, e_2, e_3, e_4, e_5, e_6, \dots\}$  where  $e_1 = (1, 2)$   $e_2 = (1, 3)$   $e_3 = (1, 4)$

$e_4 = (1, 5)$   $e_5 = (1, 6)$   $e_6 = (2, 3)$   $e_7 = (2, 4)$   $e_8 = (2, 5)$   $e_9 = (2, 6)$   $e_{10} = (3, 4)$   $e_{11} = (3, 5)$   $e_{12} = (3, 6)$   $e_{13} = (4, 5)$   $e_{14} = (4, 6)$   $e_{15} = (5, 6)$ .



Subsequently, the  $N[G]$  nbd chart of  $G(R)$  is a  $K_6$ -Complete diagram.

**End:** From the above representation, we can see that assuming  $n$  is even (odd), i.e.,  $n=2r$  ( $r \geq 4$ ) ( $n=2r+1$  ( $r \geq 2$ )), the nbd chart  $N[G]$  of  $G(R)$  is a Complete diagram.

Hypothesis 5.3.9: The ring of numbers modulo  $n$ ,  $R = \mathbb{Z}_n$ , fulfills every one of the standards in hypothesis 3.3.1.

$N[G]$  is characterized as  $[V(N[G], E(N[G]))]$ . be  $G(r)$ 's neighborhood chart,  $V(N[G])$  the vertex

set, and  $E(N[G])$  the edge set of  $N[G]$ , where  $E(N[G]) = u, v \in G$  ( $u$  and  $v$  are nearby if and provided that  $nbd(u) \cap nbd(v)$ .

$N[G]$  is then a Regular chart.

Evidence: Given  $Z_n$  as the ring of whole numbers modulo  $n$  and  $R = Z_n Z_n$ ,  $((R, +, \cdot, n))$  is a ring to such an extent that  $R = Z_n Z_n = (a, b) / a, b \in Z_n$  with expansion modulo  $n$  ' $+n$ ' and duplication modulo ' $n$ '.

In the event that  $x, y \in R$ ,  $x = (x_1, x_2)$   $y = (y_1, y_2)$  where  $x_1, x_2, y_1, y_2 \in R$ .

$x$  is currently neighboring  $y$ .

$$\square x \times_n y = 0 \text{ and } x, y \square (0, 0)$$

$$\square (x_1, x_2) \times_n (y_1, y_2) = (0, 0)$$

$$\square (x_1 \times y_1, x_2 \times y_2) = (0, 0)$$

$$\square x_1 \times y_1 = 0, x_2 \times y_2 = 0 \quad \square$$

Assuming  $x$  has an added substance reverse  $y$ ,  $x$  and  $y$  are coterminous.

Assuming  $R$  meets every one of the states of hypothesis 3.3.1,  $R$ 's diagram,  $G(R)$ , is a Complete bipartite chart.

At the point when  $n$  is prime ( $n \geq 3$ ) in  $Z_n Z_n$ ,  $G(R)$  is  $K_{n-1, n-1}$  – Complete bipartite diagram, where the vertex set is parceled into two divisions, each with  $(n-1)$  vertices.

We currently find the nbd diagram  $N(G)$  of  $G$ . ( $R$ ). According to  $N(G)$ , for each two vertices  $x, y \in G$ ,  $x$  and  $y$  are close by in  $N(G)$  iff  $nbd(u) \cap nbd(v)$ .

Since  $G(R)$  is a Complete bipartite organization with  $(n-1)$  vertices, each vertex in  $G(R)$  is neighboring  $(n-1)$  vertices yet not to  $(n-2)$  vertices.

That is, any two vertices in  $G(R)$  share no less than one adjoining vertex for all intents and

purpose.

Therefore, any crossing point of two nbd vertices isn't vacant.

Therefore, in  $N[G]$ , each two vertices of  $G(R)$  are adjoining.

Since  $G(R)$  has  $2(n-1)$  vertices,  $N[G]$  is a Complet chart with  $2(n-1)$  vertices.

5.3.8 Illustration: Let  $R=Z_n$   $Z_n$  be a commutative ring of whole numbers modulo  $n$ , where  $n \geq 3$  is a positive whole number.

**Case(i):** Let  $n=3$ . Then  $R= \{(0,0) (0,1) (0,2) (1,0) (1,1) (1,2) (2,0) (2,1) (2,2)\}$

what's more,  $G(R)$  is a  $K_{2,2}$  - Complete bipartite diagram with  $V(G(R))=\{(0,1),(1,0),(0,2),(2,0)\}$ . Now  $nbd [(0,1)] = \{(0,1),(1,0),(2,0)\}$   $nbd [(0,2)] = \{(0,2),(1,0),(2,0)\}$   $nbd [(1,0)]$

$= \{(1,0),(0,1),(0,2)\}$   $nbd [(2,0)] = \{(2,0),(0,1),(0,2)\}$ . Since  $nbd [(0,1)] \cap nbd [(0,2)] \neq \emptyset$ ,

$nbd [(0,1)] \cap nbd [(1,0)] \neq \emptyset$ ,  $nbd [(0,1)] \cap nbd [(2,0)] \neq \emptyset$   $nbd [(0,2)] \cap nbd [(1,0)] \neq \emptyset$ ,

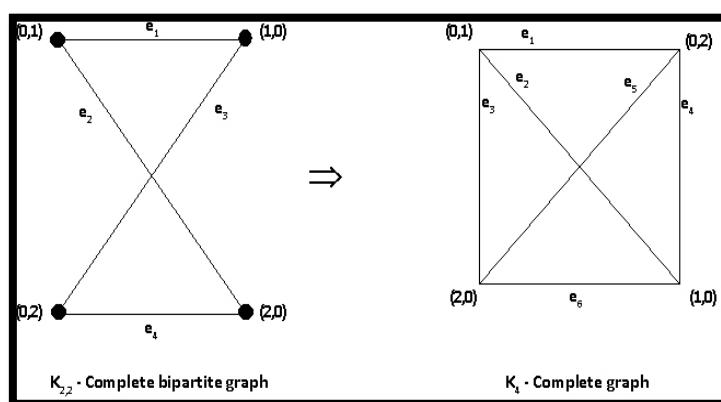
$nbd [(0,2)] \cap nbd [(2,0)] \neq \emptyset$ ,  $nbd [(1,0)] \cap nbd [(2,0)] \neq \emptyset$ . Therefore  $(0,1)$  and  $(2,0),(0,2)$  and

$(1,0)$  etc. are adjacent in  $N[G]$ . Hence  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6\}$  where  $e_1=[(0,1),(0,2)]$

$e_2=[(0,1),(1,0)]$

$e_3=[(0,1),(2,0)]$   $e_4=[(0,2),(2,0)]$   $e_5=[(0,2),(2,0)]$

$e_6=[(2,0),(1,0)]$ .



Along these lines the nbd chart  $N[G]$  of  $G(R)$  is a  $K_4$ -Complete diagram.

**Case(i):** Let  $n=5$ .

Then  $R=\{(0,0)(0,1)(0,2)(0,3)(0,4)(1,0)(1,2)(1,3)(1,4)(2,0)(2,1)(2,2)(2,3)(2,4)(3,0)$

$(3,1)(3,2)(3,3)(3,4)(4,0)(4,1)(4,2)(4,3)(4,4)\}$  and  $G(R)$  is a  $K_{4,4}$ -Complete bipartite diagram

with  $V(G(R))=\{(0,1),(0,2),(0,3),(0,4),(1,0),(2,0),(3,0),(4,0)\}$ . Now

$nbd[(0,1)]=\{(0,1)(1,0),(2,0),(3,0),(4,0)\}$   $nbd[(0,2)]=\{(0,2),(1,0),(2,0),(3,0),(4,0)\}$   $nbd[(0,3)]=$

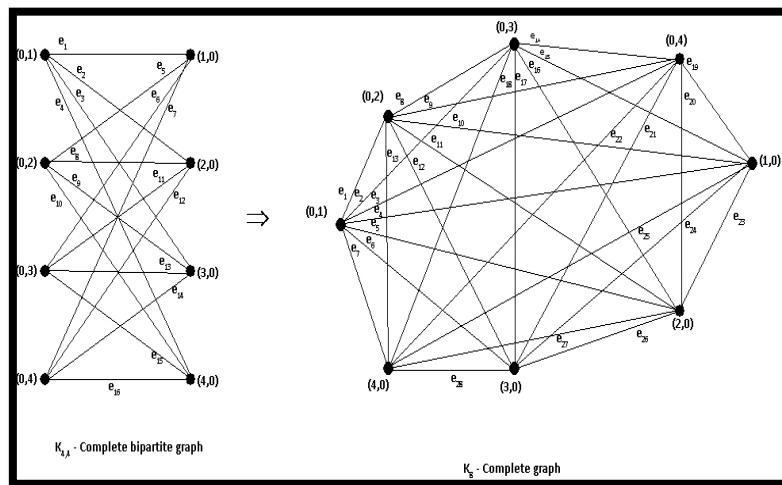
$\{(0,3),(1,0),(2,0),(3,0),(4,0)\}$   $nbd[(0,4)]=\{(0,4),(1,0),(2,0),(3,0),(4,0)\}$   $nbd[(1,0)]=\{(1,0),(0,$

$1),(0,2),(0,3),(0,4)\}$   $nbd[(2,0)]=\{(2,0),(0,1),(0,2),(0,3),(0,4)\}$   $nbd[(3,0)]=\{(3,0),(0,1),(0,2),(0,$

$3),(0,4)\}$   $nbd[(4,0)]=\{(4,0),(0,1),(0,2),(0,3),(0,4)\}$ . Since  $nbd[(0,1)] \cap nbd[(0,2)] \neq \emptyset$ ,  $nbd$

$[(0,1)] \cap nbd[(1,0)] \neq \emptyset$ ,  $nbd[(0,1)] \cap nbd[(2,0)] \neq \emptyset$   $nbd[(0,2)] \cap nbd[(1,0)] \neq \emptyset$ ,  $nbd$

$[(0,2)] \cap nbd[(2,0)] \neq \emptyset$ ,  $nbd[(0,1)] \cap nbd[(4,0)] \neq \emptyset$  etc,. Therefore  $(0,1)$  and  $(2,0),(0,2)$  and  $(4,0)$  and so forth are contiguous in  $N[G]$ .  $E(N[G])=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$  where  $e_1=[(0,1),(0,2)]$   $e_2=[(0,1),(0,3)]$   $e_3=[(0,1),(0,4)]$   $e_4=[(0,1),(1,0)]$   $e_5=[(0,1),(2,0)]$   $e_6=[(0,1),(3,0)]$  and so forth,



**End:** From the past picture, plainly in the event that  $n$  is prime ( $n \geq 3$ ),  $G(R)$  is a  $K_{n-1, n-1}$  - Complete bipartite diagram and  $N(G)$  is a  $K_{2(n-1)}$  - Complete chart.

The relationship between  $n$  and  $N[R]$  is seen in the table below.

| No of vertices ( $n \geq 1$ )( $n$ is prime) | $ R  = n^2$ | $G(R) = K_{2(n-1)}$ -Complete graph |
|--|-------------|-------------------------------------|
| 3  | 9           | $K_4$ -Complete graph               |
| 5  | 25          | $K_8$ -Complete graph               |
| 7  | 49          | $K_{12}$ -Complete graph            |
| 11   | 121         | $K_{20}$ -Complete graph            |
| .  | .           | .                                   |
| .  | .           | .                                   |
| .  |             | .                                   |

## CONCLUSION

The investigation of zero divisor diagrams of commutative rings reveals interesting associations between ring hypothesis and chart hypothesis; mathematical procedures help in the comprehension of diagram elements as well as the other way around. Beck characterized the zero divisor diagram of a commutative ring  $R$  in 1988, where the ring and two vertices  $x, y$  are close by iff  $xy=0$ . Anderson and Livingston [9,10] refreshed the thought of zero divisor charts by restricting the vertices to non-zero no divisors of the ring  $R$ . Anderson et al. [7,8,9,10, 11] explored various chart hypothetical properties of zero divisor diagrams, remembering the quantity of coteries for  $(R)$ , where  $(R)$  is the zero divisor chart of  $R$ . Numerous researchers, including N.Cordova, C.Cholston, A.Duane, V.K.Bhat, R.Raina, and others, have widely explored the zero divisor chart of the ring of whole numbers modulo  $n$ .

Abu obsa et al [ 1 ] proposed zero divisor diagrams for the ring of Gaussian whole numbers modulo  $n$  in 2008, where they investigated various chart includes and set up a few diagram spans for the diagram on Zn I ATANI, S. EBRHIMI, KOHAN, M.SHAJARI, SARVANDI, and Z. EBAHIMI examined the safeguarding of the distance across and grith of the zero divisor chart as for an ideal of a commutative ring when stretching out to a limited direct result of a

commutative ring in 2014.

Considering the previous, we examined the diagram hypothetical properties of a commutative ring  $R = Z_n$  (or)  $Z_n$  and summed up the control boundaries on  $Z_n$  charts all through this thesis. We likewise dispatched the exploration of nbd diagrams on  $R$ 's chart and found that the subsequent charts are customary, finished, and complete bipartite diagrams.

Remembering all of this, we might want to expand our exploration to incorporate different kinds of zero divisor diagrams, star charts, caylay diagrams, Euler charts, etc, of commutative rings, just as immediate results of commutative rings.

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